PROJECT/LESSON FOR GEOMETER SKETCHPAD CLASS

SUBMITTED BY CAROLYN WARD, NOV.10, 2001

The following sources were used in writing this lesson:

- "Area and Perimeter Connections", by J. B. Kennedy, Mathematics Teacher, March 1993.
- "Teaching Relationships between Area and Perimeter", by M. E. Stone, Mathematics Teacher, November 1994.
- Implementing Standards-Based Mathematics Instruction, published by Teachers College Press, 2000.

WEST VIRGINIA IGO'S COVERED IN THIS LESSON:

GEOMETRY AND APPLIED GEOMETRY IGO'S:

(The following paragraph is the introduction to the IGO's and the ideas mentioned are very much a part of this lesson.)
Introduction: "The study of geometry should include experiences activities that foster in students a feeling for the value of geometry in their lives. Students should be encouraged to develop conjectures by inductive processes using manipulatives a computer software such as Geometer's Sketchpad. Cooperative learning groups are particularly effective in allowing students to become proficient in analyzing conjectures and formulating proofs. Emphasis should be placed on applications to the work place and everyday life and on the connections to other branches of mathematics and other disciplines. Applied Geometry will use manipulatives to enhance the understanding of geometric concepts and terminology. Working in groups will allow students to analyze applications of geometry in their lives and in the work place. Concepts will be taught using laboratory activities including the use of tools such as the graphing calculator and the Geometer's Sketchpad."

G.1: represent points, lines, and planes pictorially with proper identification, as well as basic concepts derived from these undefined terms, such as segments, rays, and angles
G.2: differentiate between inductive and deductive reasoning
G.8: explore and identify properties of quadrilaterals and verify properties for parallelogram, rectangle, rhombus, square, and trapezoid.
G.15: discover the lengths of sides of polygons from given data
G.16: develop and apply formulas for area, perimeter, surface area, and volume and apply them in the modeling of practical problems
G.23: use appropriate software to practice and master Geometry and Applied Geometry instructional objectives.
G.24: use a calculator to perform operations on whole numbers, fraction, and decimals

WEB SITES

www.peda.com/poly
www.ohiomathworks.com
www.pbs.org/wgbh/nova/proof

SUMMARY OF THE LESSON

Grade levels: 8-12

Materials:
String 64 centimeters long, ends connected (be sure to cut a little more than 64 cm to allow for knot)
Pencils
Rulers
Centimeter graph paper
Calculators
Geometer Sketchpad

Number of days: 2 or 3

Objectives:
- Explore the mathematical concepts of the effect on an area when a fixed perimeter figure is altered.
- Construct and measure a model, which will allow easy comparison between different conditions.
- Employ hands-on algebraic and geometric techniques to find the dimensions yielding the maximum area.
- Ask students to make a generalized algebraic statements from data.
- Help students achieve a deeper understanding of the concepts of area and perimeter and their relationships.

PART 1 (Modeling the problem with hands-on materials):

The class begins by discussing the meanings of AREA and PERIMETER and by relating the concepts to agriculture, architecture, social studies, and our every-day world around us. The overhead "**BUILDING A PEN"** is shown and the students are divided into groups of 2 to 4. They are given 10-20 minutes to use the materials (string, pencils, rulers, centimeter graph paper, calculators) and answer the questions shown on the overhead.

The question "What shape will have the largest area for a given perimeter?" identifies an important relationship between area and perimeter. You may want to discuss with your students the following facts, which were taken from the Michael Stone article in the Mathematics Teacher article cited earlier.

> "Examples throughout the world is cylindrical houses built in West Africa, beehive houses of the Chagga people of Mount Kilimanjaro, and hemi-spherical igloos of the Eskimo people. While struggling to cope with their environments, all these cultures used available materials to build dwellings with shapes that maximized floor space. Since perimeter is limited by the available materials, determining what shape will have the largest area is very important to people who find materials difficult to work with or hard to obtain."

Each group can then present to the class their findings. The teacher can lead a discussion of the conjectures and the resulting conclusions.

PART 2 (Using Geometer Sketchpad to "build" different shapes and compare areas)

Students are given the **WORKSHEETS "RELATIONSHIPS BETWEEN AREA AND PERIMETER"** and work in pairs to fill-in the tables and conjectures. The four worksheets pose questions dealing with area and perimeter. They make conjectures and then use Sketchpad to investigate. The software allows them to draw many different polygons and to see the measures of the sides, the perimeters, and the areas simultaneously.

Students complete the tables with dimensions from the software. They can take "Snapshots" of their screen displays and save them on disk. They can print out their diagrams.

The following generalizations should become evident to them as they draw many types of figures:

- Worksheet 1 generalization: The students who worked with quadrilaterals, N=4 for example, will see that, with a constant perimeter, the area is greatest when a square is formed. Other types of polygons used will reveal that as the shape approaches a regular n-gon the area increases.

- Worksheet 2 generalization: The students discover that to minimize perimeter among quadrilaterals with the same area, the shape again must be a square. The same is true with other n-gons (congruent angles and sides will result in the least perimeter).

- Worksheet 3 generalization: (This worksheet investigates the same question as the "FENCE" question using the string) The students will see that the circle has the greatest area when all shapes and n-gons have the same perimeter/circumference.

- Worksheet 4 generalization: They discover that among all figures with the same area, a circle has the smallest perimeter.

The teacher may want the groups to share their results and discuss the class findings. The Geometer Sketchpad allows the students to discover these findings by many examples in a short period of time.

RELATIONSHIPS BETWEEN AREA AND PERIMETER

Worksheet 1

Names_____

Discovery Question 1: For all N-GONS with the same perimeter (let N be some positive integer), what shape will have the greatest area?

Conjecture: Write a statement that you think accurately answers the above question.

_____

_____

Use the table below to record the results from your Geometer Sketchpad drawings and calculations.

FIGURE (triangle,quadri-lateral,pentagon, etc.)	No. of Sides	Dimensions (nearest tenth unit)	Perimeter (cm)	Area (sq.cm)

Write a concluding true statement or generalization on the basis of your findings.

_____

_____

_____

RELATIONSHIPS BETWEEN AREA AND PERIMETER

Worksheet 2

Names_____

Discovery Question 2: For all N-GONS with the same area (let N be some positive integer), what shape will have the smallest perimeter?

Conjecture: Write a statement that you think accurately answers the above question.

_____

_____

Use the table below to record the results from your Geometer Sketchpad drawings and calculations.

FIGURE (triangle,quadri-lateral,pentagon, etc.)	No. of Sides	Dimensions (nearest tenth unit)	Perimeter (cm)	Area (sq.cm)

Write a concluding true statement or generalization on the basis of your findings.

_____

_____

_____

RELATIONSHIPS BETWEEN AREA AND PERIMETER

Worksheet 3

Names_____

Discovery Question 3: For all figures with the same perimeter, what shape will have the greatest area?

Conjecture: Write a statement that you think accurately answers the above question.

_____

_____

Use the table below to record the results from your Geometer Sketchpad drawings and calculations.

FIGURE	No. of Sides (if applicable)	Dimensions (nearest tenth unit)	Perimeter (cm)	Area (sq.cm)

Write a concluding true statement or generalization on the basis of your findings.

_____
_____
_____

RELATIONSHIPS BETWEEN AREA AND PERIMETER

Worksheet 4

Names_____

Discovery Question 4: For all figures with the same area, what shape will have the smallest perimeter?

Conjecture: Write a statement that you think accurately answers the above question.

_____
_____

Use the table below to record the results from your Geometer Sketchpad drawings and calculations.

FIGURE	No. of Sides (if applicable)	Dimensions (nearest tenth unit)	Perimeter (cm)	Area (sq.cm)

Write a concluding true statement or generalization on the basis of your findings.

_____
_____
_____

BUILDING A PEN

Mr. Jones' class is planning to build a pen to raise rabbits for their science fair. They have 64 feet of fencing to use. They are considering different shapes and dimensions of the pen that they could build.
The class discusses these questions:

- **What different SHAPES and SIZES of pens can we build with 64 feet of fencing? (Consider ANY shapes----including straight or curved sides)**
- **Will all the pens we can build provide the same LIVING SPACE for our rabbits?**
- **Or, if the shape DOES matter, what SHAPE and DIMENSIONS will give the MOST living space for the animals?"**

Today, your group is to use the materials provided (string, rulers, calculators, graph paper) and answer the above questions. Discuss the problem in your group. Organize your ideas and work on paper. Make a conjecture about your conclusions.