

EXPLORING QUADRILATERALS AND PARALLELOGRAMS

PREPARED BY
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2001

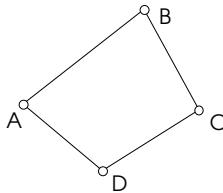
Quadrilaterals

Exploring Parallelograms

This Geometer's Sketchpad activity will investigate quadrilaterals and parallelograms which are a special type of quadrilateral.

A **quadrilateral** can be defined as 4 coplanar segments which intersect only at their endpoints (each endpoint belongs to only two segments).

Begin by opening the Geometer's Sketchpad software. Using the segment tool, construct a quadrilateral, similar to the one shown below. After you have constructed it, label the vertices using the label tool.

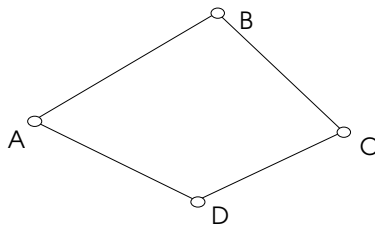


Quadrilateral ABCD is called **convex**. A convex polygon is one that no external segment can be drawn to connect any two vertices of the polygon.

Now, measure all of the interior angles for the quadrilateral you have constructed. Remember, to measure an angle, you must select the endpoints of the two segments that form the angle and then go to the *Measure* menu and select *Angle*.

Once all of the angles have been measured, go back to the *Measure* menu and select *Calculate*. Now, sum the angles for the quadrilateral.

$$\begin{aligned}m\angle DAB &= 71^\circ \\m\angle ABC &= 91^\circ \\m\angle BCD &= 83^\circ \\m\angle CDA &= 115^\circ \\m\angle DAB + m\angle ABC + m\angle BCD + m\angle CDA &= 360^\circ\end{aligned}$$



Now, compare

your measurements

to those of your classmates.

What did you

discover? _____

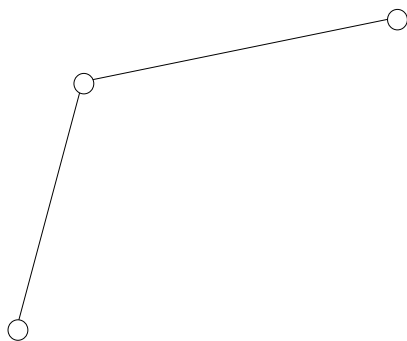
By selecting one of the vertices using the select tool, the shape of the quadrilateral can be changed. As you change the shape, the computer will recalculate the angle measurements. You should notice that for any convex quadrilateral, the sum of the angles will equal 360° . Concave quadrilaterals have the same property, however, Geometer's Sketchpad will not show this with the quadrilateral you constructed due to the way it measures angles.

Select a new Sketchpad window by selecting *File* and then *New Sketch*. We are now going to explore the world of **parallelograms**. A parallelogram is a quadrilateral where opposite sides are parallel. Parallelograms have several properties that we can discover using Geometer's Sketchpad.

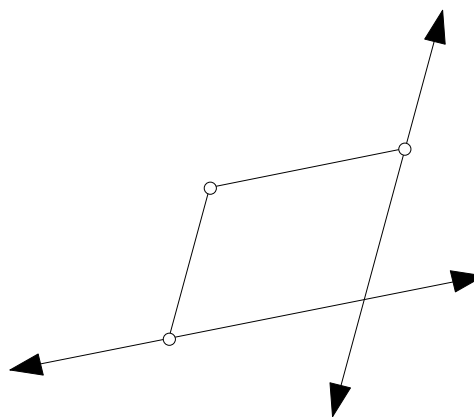
First, we need to construct a parallelogram. It is important that it be constructed in a precise manner so that no matter how we change the shape, it will remain a parallelogram.

Step 1: Use the segment tool to construct two segments connected at one endpoint.

Step 2: Use the select tool to select one side and the endpoint of the other segment (the endpoint that the two segments do not share). Go to *Construct* and select *Parallel Line*. Now, use the same process and select the other segment and the endpoint. Once again select *Construct* and *Parallel Line*.

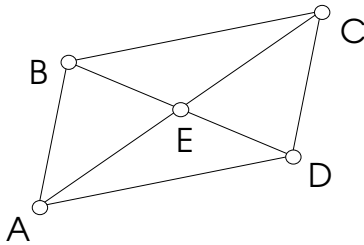


Step 1



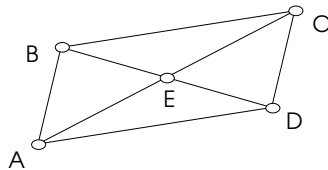
Step 2

Step 3: Where the two lines intersect, use the point tool to place a point. Next, select the two lines and go to the *Display* menu and choose *Hide Lines*. Now use the segment tool to connect the endpoints of the two segments to the other point to form a parallelogram. Use the segment tool to construct the two diagonals and place a point at their intersection. Finally, use the label feature to label the vertices as well as the point of intersection of the diagonals.



Step 4: Using the *Measure* menu and the selection tool, measure the length of each side, the measure of each interior angle and the measurements of each diagonal

$m\angle BAD = 68^\circ$	$m\overline{AD} = 0.9$ inches
$m\angle BCD = 68^\circ$	$m\overline{BC} = 0.9$ inches
$m\angle ABC = 112^\circ$	$m\overline{DC} = 0.5$ inches
$m\angle CDA = 112^\circ$	$m\overline{AB} = 0.5$ inches



BE = 0.4 inches
 ED = 0.4 inches
 AE = 0.6 inches
 EC = 0.6 inches

Again, the measurements for your quadrilateral will likely vary from those in the diagram above. If you examine the measurements, you will notice that some of the measurements are equal. Geometer's Sketchpad allows us to change the dimensions/shape of the quadrilateral while keeping the shape a parallelogram. Try selecting one of the vertices and dragging your mouse to change its size/shape. As you do this notice what happens to the measurements. What properties of parallelograms can you conjecture from this exercise? Use the following page to write your answer.

1. Opposite angles of a parallelogram are congruent
2. Opposite sides of a parallelogram are congruent
3. Consecutive angles of a parallelogram are supplementary
4. The diagonals of a parallelogram bisect each other.

BRAINSTORMING: (Group Work)

The last part of this activity will involve investigation of the diagonals of a parallelogram. Construct a parallelogram and calculate the total length of each diagonal as well as the angles formed by the intersection of the diagonals. In addition, you will need to find the length of each side as well as the measure of the interior angles.

Use the drag feature of Geometer's Sketchpad to analyze (length of side / interior angles) parallelograms when:

1. The diagonals are perpendicular.

2. The diagonals are congruent.

3. The diagonals are both perpendicular and congruent.

RHOMBUS - A parallelogram where the

diagonals are perpendicular

**RECTANGLE - A parallelogram where the diagonals are congruent
All interior angles equal 90°**

**SQUARE - A parallelogram where the diagonals are perpendicular and congruent
All sides of a square are congruent
All interior angles equal 90°
A square is both a rhombus and a rectangle**

WEB RESOURCES

Home of Geomter's Sketchpad (Key Curriculum Press)

<http://www.keypress.com/sketchpad/>

Geomter's Sketchpad Resources

<http://mathforum.org/sketchpad/sketchpad.html>

A First Glance at Quadrilaterals

<http://www.math.com/school/subject3/lessons/S3U2L3GL.html>

Advanced Quadrilateral Activities with Geometer's Sketchpad

http://www.mdabbs.fslife.co.uk/Maths_Pages/SketchPad_Files/Quadrilaterals/Quadrilaterals.html

A Web Simulation Activity to Discover Quadrilaterals

<http://www.mste.uiuc.edu/m2t2/geometry/quadsDiag.html>