

Blue Ribbon Workshop Project: The World Series Problem
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The following activity is appropriate for various mathematics classes. Statistics and Probability skills are currently tested at each level (grades 9 – 11) with SAT9, whether or not students are actually in a statistics course. The following objectives then, are from the different levels of the SAT9 as well as the different courses where students might use this activity.

SAT9 Objectives:

Grade 9: 29. Make a prediction from a statistical sample
32. Predict outcomes of a compound event, given a theoretical probability
33. Estimate probability, given experimental data or a graph

Grade 10: 34. Estimate probability, given experimental data or a graph

Grade 11: 33. Predict outcomes of a compound event, given a theoretical probability
34. Estimate probability, given experimental data or a graph

Content Standards and Objectives

Algebra 1: Al.2.20 predict the outcomes of simple events using the rules of probability.

Applied Math 1:

AM1.2.16 predict the outcomes of simple events using the rules of probability.

Conceptual Math:

CM.5.3 determine possible outcomes using tree diagrams and the counting principles of permutations and combinations.

CM.5.4 apply the basic probability rules in expressing the chances of events occurring using technology when appropriate.

Probability and Statistics:

PS.5.1 distinguish between experimental and theoretical probability.

PS.5.3 determine possible outcomes using tree diagrams and the counting principles of permutations and combinations.

PS.5.4 express the chances of events occurring either in terms of a probability or odds.

The World Series Problem:

Each fall, the winners of the National League and American League meet in the World Series. Whichever team wins four games out of a possible seven will become World Champions (in at least the North American World.) This activity will attempt to predict how many games it will take for one team to win the series. There are many factors that we will ignore: weather, injuries, home field advantage, etc. We will simplify the problem by choosing the probability that the National League (NL) team will win an individual game. We will use this same probability throughout the simulation to predict the number of games necessary to win (or lose) the series.

The simplest version:

We'll start with the chance of the NL team winning at 50%. We'll use the random integer generator to generate 7 numbers; each being 0 for a loss or 1 for a win.

The syntax on the TI-83 Plus is `randint(0, 1, 7)`. The keystrokes: MATH, PRB, `randint`, then fill in the arguments for the function. Fill in the table below with your results for ten years of World Series. For example, if one result is 1 0 1 1 0 1, then it took 5 games to win the series, and the last two games would not have been played. For each trial, count how many games were required to determine the series.

Trial #	# of games required
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Average the number of games required to win the series: _____
 This average is an experimental estimate of the *expected value*.

You will probably get a better answer if you have many more experiments. On the blackboard, list your average along with the results of your classmates. Find the average of all averages. _____

Now let's look at the problem analytically.

The following table was adapted from one provided by Jay Hill at <http://www.mste.uiuc.edu/hill/ev/seriessol.html>. To find the required combination values, use your calculator. The keystrokes for C(n, r), starting on the home screen are: type in the number for n, then MATH, PRB, nCr, type in the r value, then enter. Complete the chart below.

# of games to win the series	explanation	probability
<u>1</u>	The series must take more than 1 game, because the champion must win 4 games.	0
<u>2</u>	The series must take more than 2 games, because the champion must win 4 games.	0
<u>3</u>	The series must take more than 3 game, because the champion must win 4 games.	0
<u>4</u>	In order for the series to end after four games, one team must win the first four games in a row.	$(1/2)(1/2)(1/2)(1/2) + (1/2)(1/2)(1/2)(1/2) = 2/16 = 1/8 = 0.1250$
<u>5</u>	In order for the series to end in 5 games one team must win exactly 3 out of the first 4 games (in any order) and then win the fifth game.	$C(4,3) (1/2)(1/2)(1/2)(1/2)(1/2) + C(4,3) (1/2)(1/2)(1/2)(1/2)(1/2) =$
<u>6</u>	In order for the series to end in 6 games one team must win exactly 3 out of the first 5 games (in any order) and then win the sixth game.	$C(5,3) (1/2)(1/2)(1/2)(1/2)(1/2)(1/2) + C(5,3) (1/2)(1/2)(1/2)(1/2)(1/2)(1/2) =$
<u>7</u>	In order for the series to end in 7 games one team must win exactly 3 out of the first 6 games (in any order) and then win the seventh game.	$C(6,3) (1/2)(1/2)(1/2)(1/2)(1/2)(1/2)(1/2) + C(6,3) (1/2)(1/2)(1/2)(1/2)(1/2)(1/2)(1/2) =$

You may check your answers to make sure that the probabilities add to 1. Do they?

To find the expected value, multiply the number of games needed by its probability.

Expected value =

Was your answer close to the experimental value?

Do you expect it to be exactly the same?

A more advanced case:

This time let's suppose that we believe one team has a better chance of winning. We can set this probability to be anywhere between 0 and 1. (Why?) For our example, let's define the probability that the NL team will win any game at 3/5. We will use the Probability Simulation Application available for the TI-83Plus. This application can be downloaded from www.ti.education.com for free. Tossing a coin will determine which team wins a game: tails, NL wins, heads, AL.

NOTE: The "soft menu" is located on the bottom of the screen, corresponding to the F1 – F5 keys on the calculator. The choices on the soft menu change, dependent on the application being used.

To open the application, press the APPS key, then scroll down to find and select Prob Sim, enter, then press any key to start the application. We will flip a coin to simulate the win or loss of a game; since our assumption is that NL has more of a chance to win, we will weight the coin toss.

Choose 1) Toss Coins, ENTER. Choose SET to set the options:

Trial set: 7 Coins: 1 Graph: Freq StoTbl: All ClearTbl: Yes

Press ADV (the F2 key) to set the probability of each option: .60 for tails, .40 for heads, then OK.

OK to get back to the simulation.

Press TOSS on the soft menu; TABL to see the results of the tosses in a table. The number of games required to win the series is the number of the first toss where there has been 4 heads or 4 tails. Fill in the chart below to find your result for ten series.

Trial #	# of games required
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Find the average of your ten series to find an estimate of expected value: _____

Combine your results with others in your class to get a better estimate: _____

Did you get the same answer as when the teams had an even chance to win? _____

To exit the program, the following keystrokes should take you back to the home screen: esc, YES, QUIT, YES.

Fill in a chart similar to the analysis for the “even chances” case to find the expected value of the number of games needed to end the series. This time, individual probabilities will be 3/5 and 2/5.

# of games to win the series	explanation	probability
<u>1</u>	The series must take more than 1 game, because the champion must win 4 games.	0
<u>2</u>	The series must take more than 2 games, because the champion must win 4 games.	0
<u>3</u>	The series must take more than 3 game, because the champion must win 4 games.	0
<u>4</u>	In order for the series to end after four games, one team must win the first four games in a row.	$(3/5)(3/5)(3/5)(3/5) + (2/5)(2/5)(2/5)(2/5) =$
<u>5</u>	In order for the series to end in 5 games one team must win exactly 3 out of the first 4 games (in any order) and then win the fifth game.	$C(4,3) (3/5)(3/5)(3/5)(2/5)(3/5) + C(4,3) (2/5)(2/5)(2/5)(3/5)(2/5) =$
<u>6</u>	In order for the series to end in 6 games one team must win exactly 3 out of the first 5 games (in any order) and then win the sixth game.	$C(5,3) (3/5)(3/5)(3/5)(2/5)(2/5)(3/5) + C(5,3) (2/5)(2/5)(2/5)(3/5)(3/5)(2/5) =$
<u>7</u>	In order for the series to end in 7 games one team must win exactly 3 out of the first 6 games (in any order) and then win the seventh game.	$C(6,3) (3/5)(3/5)(3/5)(2/5)(2/5)(2/5)(3/5) + C(6,3) (2/5)(2/5)(2/5)(3/5)(3/5)(3/5)(2/5) =$

Check your probabilities by adding. Do they add to 1? If not, go back and check your math.

To find the expected value, multiple the number of games needed by its probability.

Expected value =

Was your answer close to the experimental value?

Internet Sites for Activities

Brewer, W., et al. “The Probability Dilemma.” Version 2. 15 Aug 2002 . [Texas Instruments Explorations© and Free Resources](http://otresources.ti.com/otr/user/ActivityView.do?activityId=875&mode=view). 12 Oct 2002 <<http://otresources.ti.com/otr/user/ActivityView.do?activityId=875&mode=view>>.

This activity looks at all possible combinations of male and female foals if a mare has five foals. The theoretical probability of different female/male combinations is then given.

Hill, J. “How Many Games Does it Take to Win the World Series?” MSTE, University of Illinois. 12 Oct 2002 <<http://www.mste.uiuc.edu/hill/ev/seriesprob.html>>.

My activity is based on the simulation suggested at this site. An analysis is then given about how to solve the problem using combinations. I used the charts from this site with permission, via e-mail, from Jay Hill.

Reese, George. "The Birthday Problem: A short lesson in probability." 9 Nov 2000. MSTE, University of Illinois. 12 Oct 2002 <<http://www.mste.uiuc.edu/exner/java.f/birthday/default.html>>.

This site simulates the birthday problem for various sizes using an applet. It then links to an explanation of the problem with a probability distribution graph.

"What are the Odds?" 18 Oct 2002. Productivity in the Classroom. Microsoft Education. 18 Oct 2002 <<http://microsoft.com/education/default.asp?ID=WhatOdds>>.

This lesson provides an introduction to probability theory, a branch of mathematics that everyone can relate to. It has real-world applications at every level of complexity, from the simple odds of flipping a coin to the complexities of game theory.