Lesson Plan
Using experimental probability to estimate population probability

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I. Objectives:

Students will calculate experimental probability through simulations, estimate population probability, and graph the results using the TI-83.

II. Student Goals:

Students will be able to perform probability simulations using manipulatives and the TI-83.

III. West Virginia Instructional Goals and Objectives:

This activity is designed for use in an Algebra I class and meets the following WV IGO's.

AGP 4,5,6,21
AI 6,8,21
PS 1,2,4,19,20

IV. Prior Knowledge:

Students have limited knowledge of basic probability concepts and have a basic working knowledge of graphing calculators.

V. Procedures:

1. Review probability formula-

Probability of an event, **A**, made up of equally likely outcomes,

$$P(A) = \frac{\#\text{of favorable outcomes in A}}{\#\text{of possible outcomes}}$$

2. Present problem:

Joe is a pretty good free-throw shooter. In fact, he typically misses only 1 shot out of six. During most basketball games, Joe shoots four foul shots. What is the probability (assume all shots are independent and not influenced by other "game" factors) that Joe will make all four foul shots in a game?

3. Conduct simulations

Items needed:
Students in groups of two
A six-sided standard number cube for each group
Basketball simulations worksheet (attached)
Graphing calculators

Generation procedure:
To generate the data we will be rolling a 6 sided number cube. If a 1 is rolled then this is considered a miss. If a 2,3,4,5,or 6 is rolled then the foul shot has been made. The students need to roll the cube 4 times to simulate shooting 4 foul shots (This is considered one trial). If they roll a 1 during one of these 4 shots then they have failed to meet the desired outcome. To generate a large enough data sample each student will have 5 sets of data, they will roll the cube 20 times. Remember we are only looking for yes's. We will count the number of yes's and divide by the total number of trials. This is p^ (p hat). P hat from each group will be recorded on the collection sheet and entered into the TI-83.

4. Collect simulation data

Following the set up on the simulation data chart, collect student simulation data and complete chart with students.
Demonstrate with TI -83 the generation of random sets of data with defined parameters. Complete the data collection chart so that the cumulative frequency (# of games) totals 100.

Keystrokes:
MATH-PRB-5randInt(1,6,4)-ENTER will generate sets of random numbers from 1 to 6 in groups of four. Note: when randInt(is displayed the calculator is looking for a,b,c where a and b are the range of possible numbers and c is the number of foul shots. Once this is set up the students will hit the enter key 5 times to simulate 5 rolls of the cube. They can clear the screen after each 5 entries to start with a fresh screen for each trial.

5. Graph Data

A. Line graph, Enter data cumulative frequency (# of games) into TI-83 as L_1 and relative frequency (probability) as L_2.

Keystrokes:
STAT-EDIT1-ENTER-scroll to L_1 and L_2 and enter data.

Plot a line graph of the relationship between the cumulative frequency (# of games) and the relative frequency (probability) on the TI-83.

Keystrokes:
2^{nd}-Y=(STAT PLOT)-1-ENTER-turn on Plot 1-choose line graph icon (second choice)-set Xlist as L_1 and Ylist as L_2

set WINDOW keystrokes: WINDOW- Xmin=0 – Xmax=100 – Xscl=5 – Ymin=0 – Ymax=1 – Yscl=.1 – Xres=1 – GRAPH

If desired extend data to these graphs:

B. Histogram
Keystrokes:
STAT-EDIT-scroll to L_4 and enter values of p^
2^{nd}-Y=(STAT PLOT)-2-ENTER-turn on Plot 2-choose histogram icon-set Xlist as L_4 . Set WINDOW –Xmin=.2 – Xmax=1 – Xscl=.1 – Ymin=0 – Ymax=20 – Yscl=5 – GRAPH

C. Box plot
Keystrokes:
2^{nd}-(STAT PLOT)-3-ENTER-choose box plot icon-set Xlist as L_4
To superimpose the box plot onto the histogram choose plot 1 and plot 2 both on.

6. Discussion of Results

A. Probability Rules and Concepts
P^(experimental probability)= $\frac{\text{\#of times the outcome occurs}}{\text{\#of times the experiment is repeated}}$

Law of Large Numbers – The Law of Large Numbers states that sample averages of random quantities approach the true populations mean as the sample size increases.

Population probability (p) = P(A)= #of favorable outcomes in A

#of possible outcomes

Probability multiplication rule – if A and B are independent events, the P(A and B)= P(A)*P(B)

B. Connection

Experimental probability (p^) is an estimate of the true population probability (p) with greater accuracy as the number of trials increases.

By graphing the relationship between the cumulative frequency and the relative frequency of the experimental data, discuss with students the "settling down" of the relative frequency (the probability of the event occurring) on the line graph.

*** "p^" is approximating "p"

Next, apply the probability law for independent events. Show that this will give the true population probability.

Calculate p=5/6*5/6*5/6*5/6=5/6́; since the probability of making a free throw is 5/6 and we are trying to make four shots in a row (in a game).

Compare the calculated populations probability (p) with the estimated population probability (p^) as the number of games (cumulative frequency) became large. Discuss.

VI. Resources used:

1. Blue Ribbon Data Analysis and Probability Institute manual by James Lang of Valencia Community College

2. Intro Stats Preliminary Edition by Richard D. Deveaux and Paul F. Velleman copyright 2003, Addison Wesley

VII. Activity Sources

1. The lesson plan was created from a problem in the activity "Babies and Basketballs" in the Problems with a Point: November 15, 2001 collection and then modified and extended. The web address for this site is: www2.edc.org/mathproblems/listProblem.asp

2. http://askeric.org/cgi-bin/lessons.cgi/Mathematics/Probability

3. www.shodor.org/interactivate/activities/spinner/index.html