Tesselations – Art and Mathematics

Time: In class -- 90 minutes for tessellation and another 90 for assessment
Outside of class 30 to 60 minutes

Prerequisites: Transformational geometry, symmetry, and polygon angle sum

Materials: Pattern blocks or
Cuisenaire Polygons or
Geometry Lab Drawing Template and
Geometer's Sketchpad

Strategies: Cooperative Learning Groups Rally Table
Numbered Heads Together
Pair Share

CSO's: ***G.3.14:*** Develop properties of tessellating figures and use those properties to tessellate the plane

Resources:
http://members.cox.net/tessellations/index.html
http://www.geom.uiuc.edu/~demo5337/Group4/Tsslatns.html
http://mathcentral.uregina.ca/RR/database/RR.09.96/archambl.html
http://www.math.byu.edu/~lfrancis/readings302/Tesselation_with_CWS.html
http://ccins.camosun.bc.ca/~jbritton/jbsymteslk.htm

101 Project Ideas for Geometer's Sketchpad by Key Curriculum Press

Assessment:

Design an entry hall floor for a new conference center that tessellates but does NOT use squares.

Students are required to design and create a scale version of the T shaped floor using a ruler and protractor, or Geometer's Sketchpad, or pattern blocks. The design must be colored with colored pencils, markers, crayons, or colored printer. The design must be accompanied by a bid proposal including location(s), materials, costs, and explanation.

To complete the bid proposal students will need at least 30 minutes of Internet time to research materials and labor costs or ask a local building supply business to speak to the students about materials and labor cost.

Lesson:

Polygon Manipulation Activity

Teacher instructions:

Students will be working in Rally Table cooperative groups of 4. "Rally Table" means each student will be working on related activities. In this case each student will be trying to create a tessellation using one polygon. The students in the groups will need to work as "Numbered Heads Together" because they will have to come to an agreement that the individual's conjecture is true. The process will be repeated for a 2nd polygon or using two or more polygons at the same time. Students will then answer the questions below in complete sentences.

1st Conjecture
Student #1 will try to tessellate squares
Student #2 will try to tessellate equilateral triangles
Student #3 will try to tessellate regular hexagons
Student #4 will try to tessellate regular pentagons

All students in the group will try to tessellate octagons.
Answer the following questions:

1. **What is the sum of the measures of the angles of your regular polygon?**
2. **What is the measure of each angle of your regular polygon?**
3. **What is the sum of the angles at each vertex of your tessellation?**

Each student will write a conjecture concerning why a polygon tessellates. The group will discuss each of the individual conjectures and make a group conjecture. Now students will test the conjecture with the next polygons.

2nd time
Student #1 will tessellate rhombus 120-60
Student #2 will tessellate rhombus 165-15
Student #3 will tessellate squares and trapezoids
Student #4 will tessellate hexagons and triangles

Answer the questions with complete sentences and remember to explain why.

Did your conjecture work? Why or why not?
What other regular polygons will tessellate and why?
Why will pairs or groups of polygons tessellate?

Print off a copy of the student worksheet for each student.

Polygon Manipulation Activity

You are to work in your Rally Table group. Each person in the group is responsible for a different polygon or group of polygons. You are to tessellate the plane below with your assigned polygon(s). To tessellate means to cover the plane with no holes or overlap of polygons.

1st - **Student #1 will try to tessellate squares**
Student #2 will try to tessellate equilateral triangles
Student #3 will try to tessellate regular hexagons
Student #4 will try to tessellate regular pentagons
All students will try to tessellate regular octagons.

Answer the following questions:

4. **What is the sum of the measures of the angles of your regular polygon?**
5. **What is the measure of each angle of your regular polygon?**
6. **What is the sum of the angles at each vertex of your tessellation?**

Each student will write a conjecture (*a statement you believe is true*) as to why a polygon tessellates the plane or not. *A regular polygon that tessellates the plane creates a Regular Tessellation.*

Individual conjecture:

The group will discuss each of the individual conjectures and make a group conjecture.

Group conjecture:

Now you will test the conjecture with the next polygons.

2nd **Student #1 will tessellate rhombus 120-60**
Student #2 will tessellate rhombus 165-15
Student #3 will tessellate squares and trapezoids
Student #4 will tessellate hexagons and triangles
All students will try to tessellate with two or more polygons.

Does your previous conjecture still apply? Why or why not?

What is your final conjecture that you believe is the reason polygons tessellate a plane?

Discuss the tessellations and why tessellations work or do not work, and then answer the questions below in complete sentences. If you are having difficulty, examine the vertices of the tessellation.

What regular polygons will tessellate and why?

What regular polygons do not tessellate the plane and why?

Why will pairs or groups of polygons tessellate the plane?

Explain how the measure of the angles of each polygon relates to tessellations.

What application to real life do tessellations have?

Geometer's Sketchpad Activity

Adapted from "Tumbling Block Design" from Exploring Geometry with The Geometer's Sketchpad 1993 Key Curriculum Press page 103-105

Students will be grouped in pairs from their Numbered Heads Together group from polygon activity

Geometer's Sketchpad:

- **To start, click display menu, then preferences and select label all points or use the labeling tool to label points as you go**
- **Remember to hold down shift to select more than one object, if you do not have the updated version of Geometer's Sketchpad.**
- **If you make a mistake, just go to EDIT and undo.**
- **The points may not have the same names, that is OK, but if you want the same names you can change using DISPLAY and RELABEL POINTS**

Step 1: Using the point tool, draw points A and B.

Step 2: Select the segment selection tool; hold down the segment until you see your other choices, click on the ray.

Step 3: Using the Arrow selection tool, click on point B and holding down the shift key, click on A. Using the Construct menu, construct ray BA

Step 4: Click in white space, click on point A and holding down the shift key, click on point B. Using the construct menu, construct circle by center and point. Click in white space.

Step 5: Hold down shift and click on the circle, not on B, and click on ray BA, using the Construct menu, construct point of intersection. You should now have point C. (*You should see a circle with radius AB and a point C on the circle, making the diameter of the circle BC.*)

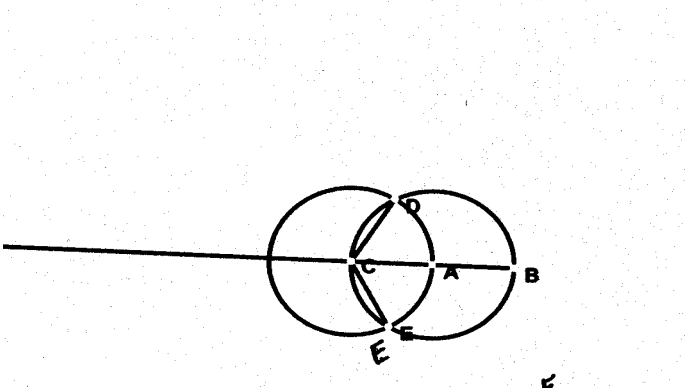
Step 6: ***(****C should be highlighted already)* so, hold down the shift key, and click on A, using the Construct menu, construct circle by center and point. (*You should now have two circles – one with center A and another with center C) Click in white space.*

Step 7: Click on circle A, hold down the shift key and click on circle C, using the Construct menu, construct point of intersection. *(The points should be labeled D and E)*

Step 8: Using the ray selection tool, hold down until you see the other choices, then click on segment

Step 9: Click in white space, click on point D (one of the intersections you just created) and holding down the shift key, click on point C. Using the construct menu, construct a segment CD. (*This segment is a chord of circle A) Click on white space on the sketch.*

Step 10: Using the arrow selection tool, click in white space, click on point E (one of the intersections you just created) and holding down the shift key, click on point C. Using the construct menu, construct a segment CE. (*This segment is a chord of circle A) Click on white space on the sketch.*



Step 11: Using the Arrow selection tool, select point B with the shift key held down, click on point A.

Step 12: Using the Construct menu, construct circle by center and point. (*You should now see three intersected circles, one with center A and two that go through point A) Click in white space.*

Step 13: Using the arrow selection tool, with the shift key held down, click on the circle A (the circle you created first) and on circle B (the one you just created), then using the Construct menu, construct point at intersection. (This is should be points F and G)

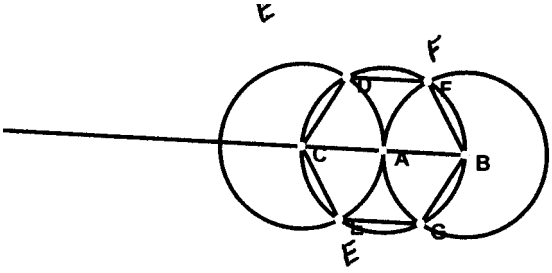
Step 14: Hold down the shift key and click on point B and point F. Using the construct menu, construct segment (this segment is another chord of circle A, click in a white space on the sketch)

Step 15: Using the ray selection tool, hold down until you see the other choices, click on segment. Hold down the shift key and click on point B and point G. Using the construct menu, construct segment (this segment is another chord of circle A, click in a white space on the sketch)

Step 16: Click in white space, click on point D and holding down the shift key, click on point F. Using the Construct menu, construct segment.

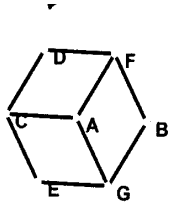
Step 17: Click in white space, click on point E and holding down the shift key, click on point G. Using the Construct menu, construct segment.

You should see three intersecting congruent circles and a recognizable polygon should be inscribed in the middle circle.



Step 18: Click in the white space, then with the shift key held down, select each circle. Using the Display menu, hide circles. Select Ray BA , using the Display menu, hide ray.

Step 19: Using the arrow selection tool, with the shift key held down, select points C and A, using the construct menu, construct segment.



Step 20: Repeat step 19 for segments AG and AF.

Step 21: Select the four points for a face such as CAGE and using the construct menu, construct polygon interior. Color it. **Repeat for the other faces.**

Step 22: Select points C and F in order, or whatever point you have on the TOP face from left to right, and using the Transform menu, Mark VECTOR.

Step 23: Select All points (to select all, use the Edit menu and select all, and then using the Transform menu, Translate by vector CF. (You should now have two "boxes")

Step 24: Translate at least 2 more times.

Step 25: Select points C and G with the shift key held down, and using the Transform menu, Mark vector.

Step 26: Select All points, then using the Transform menu, Translate by vector AG. (You should now have two rows of "boxes")

Step 27: Using Transform Menu, mark DF' (or the vertical segment from the top point on the top "box" to the vertex of the "box" underneath it) as vector.

Step 28: Select ALL and translate the two rows of blocks by vector DE' to create four rows total.

Step 29: Hide all the points

Answer the questions on the next page in complete sentences.

Describe what you see include the shapes that make up the design and the optical effect of the design.

Why do you obtain a hexagon from intersected circles?

What in nature does this tessellation model?

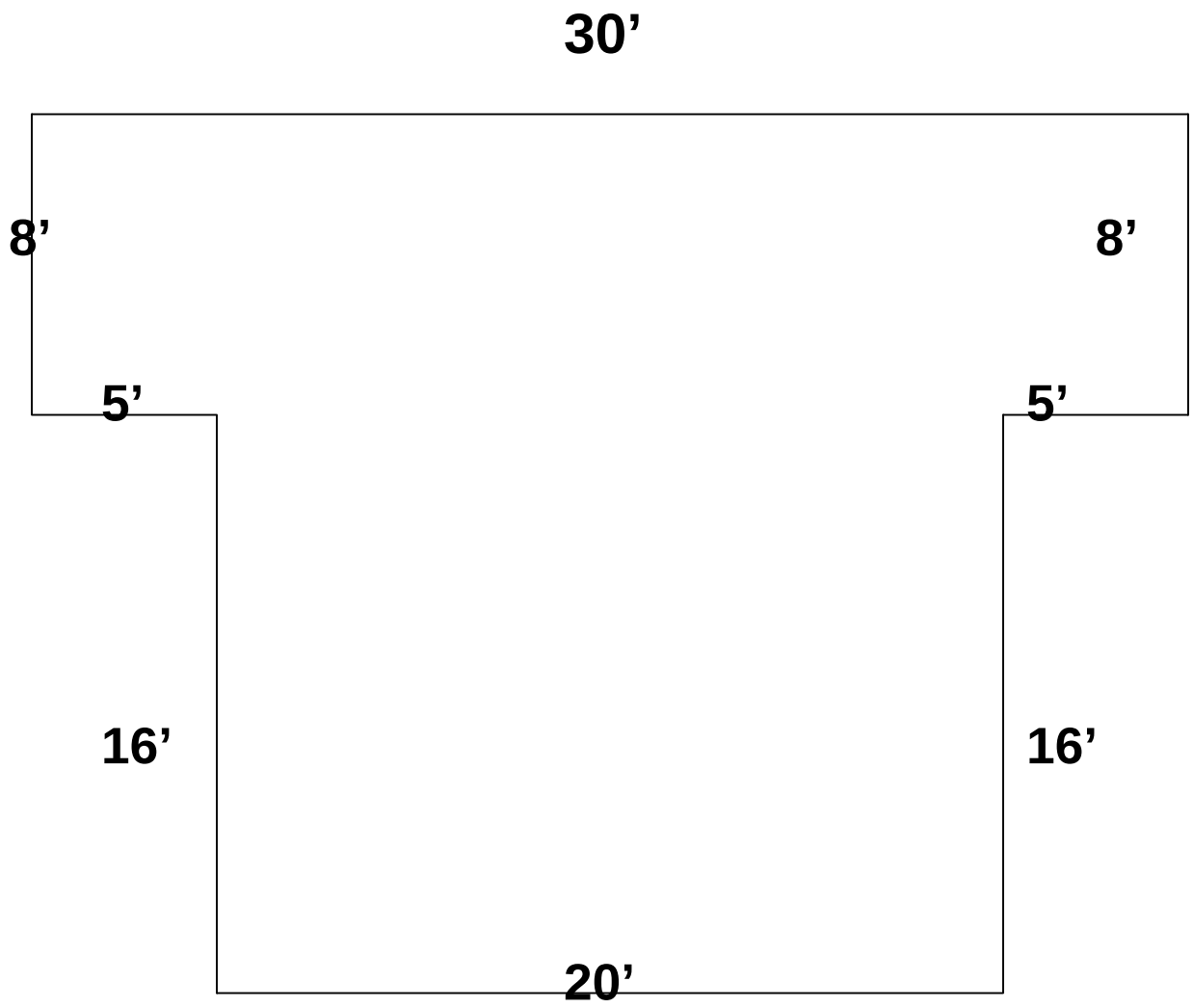
Why does this polygon tessellate the plane?

Why does any polygon that tessellates the plane do so?

Assessment:

Design an entry hall floor for a new conference center that tessellates but does NOT use squares.

1. **Design and create a scale version of the T shaped floor using a ruler and protractor, or Geometer's Sketchpad, or pattern blocks.**
2. **The design must be colored with colored pencils, markers, crayons, or colored printer.**
3. **The creation must be accompanied by a bid proposal, which will include:**
    - **Why is your creation is the best?**
    - **What makes it a tessellation?**
    - **What materials will you use and what location(s) have you chosen for your convention center?**
    - **What would be the approximate cost of the materials and labor?**
    - **Why are the materials you have chosen the most cost effective?**



To find out what is expected read the rubric on the next page.
Rubric for Assessment Activity

SCORE	Tessellation Physical example	Uniqueness Creativity	Color/ Pattern	Cost Effectiveness Description	Materials/ Convention Center Placement	Typed Bid Explanation
Distinguished	**The pattern does not create squares nor are there squares used in the tessellation and vertices correct**	**Unique pattern with creativity within each polygon**	**Every part of T floor shape neatly colored The pattern is appropriate for the locations chosen**	**Costs Included labor, materials, possible changes, and the costs were accurate.**	**Different convention center placements were considered and described with different materials for each placement**	**Description Complete, Well written, Spell Checked At least 3 location Possibilities Are explained**
Mastery	**No use of squares but the pattern does create a square and every vertex 360**	**Unique pattern but each polygon not creative**	**Every part of T floor shape neatly colored with a pattern**	**Costs included were accurate and included labor and materials**	**A placement of the center is considered and at least two possible materials are described**	**Description Complete, Well written, Spell Checked At least 1 Possibility described**
Partial mastery	**No use of squares themselves but there are clearly squares created within the pattern and vertices correct**	**Part of pattern unique and another part can be found in other places**	**The color pattern is not apparent, but all parts colored**	**Costs were explained but not Completely accurate**	**A placement of the center is considered and materials are described**	**Description Incomplete But no Spelling or Grammar Errors 1 possibility described**

Novice	**Vertices not consistently 360 and squares can be clearly seen in the pattern**	**Pattern taken from another place or person**	**The T floor pattern has some color but not completed**	**Costs were mentioned**	**A placement of the center is considered and a material is mentioned**	**Description Incomplete With Spelling And grammar Errors** **1 location possibility described**

The above are the expectations of this project assessment.