

Tessellations Using Geometer's Sketchpad

Blue Ribbon Applied Geometry Workshop
Math 693

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Introduction:

Students will follow activity sheets to create tessellations using Geometer's Sketchpad software. The lesson was adapted from activities in Exploring Geometry(1999, Key Curriculum Press). The labs are written for Version 3 of Geometer's Sketchpad. Some instructions would change for later versions. In Lab 1, a triangle is rotated to form a tile composed of two congruent triangles. By marking appropriate vectors and translating the tile, the computer screen is then tessellated. Activity sheet questions review geometric terminology and relationships. In Lab 2, a translation tessellation is created using an irregular polygon tile. The irregular tile is formed by first constructing a parallelogram and then forming and translating irregular edges between pairs of parallel sides. Extension activities give students the opportunity to produce tessellations which resemble familiar figures, to animate tessellations, and to tessellate with a tile created from a hexagon.

West Virginia Content Standards and Objectives:

Standard 3: Geometry (MA.S.3)

Students will:

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations; and
- solve problems using visualization, spatial reasoning, and geometric modeling through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics.

Objectives:

Students will:

- G.3.14 develop properties of tessellating figures and use these to tessellate the plane.
- G.3.18 using transformational geometry, create a reflection, translation, rotation, glide reflection, and dilation of a figure; and apply transformations and use symmetry to analyze mathematical situations.

Materials needed:

computer with Geometer's Sketchpad installed for each student
student activity sheets for Lab 1 and Lab 2
introduction sheet for transformations and tessellations
one floppy disk per student

Time required:

one ninety minute block period

Prior knowledge:

Students should have previous experience with Geometer's Sketchpad.

Students should have some knowledge of basic polygons and their characteristics.

Students should be able to read and follow a set of written instructions.

Students should be able to save computer files in a desired location.

Procedures:

1. Distribute student activity sheets and the introduction to tessellations.
2. Have students read the tessellation introduction either orally or silently.
3. Distribute the floppy disks to students.
4. Discuss how student work will be graded with students.
5. Students complete Lab 1 and then Lab 2 by following the activity sheets.
6. Students attempt to complete the Explore More extensions.

Evaluation:

Students will receive a grade based upon their completed tessellations and lab sheets.

The grade will be referenced to performance descriptors developed for the labs.

Performance Descriptors:

- **Distinguished**

The student demonstrates exceptional and exemplary performance with distinctive and sophisticated application of knowledge and skills that exceeds the standard in **geometry**. The student completes satisfactory tessellations for both labs and goes beyond by successfully completing at least two of the Explore More activities. The student develops outstanding written explanations for all questions on the lab sheets.

- **Above Mastery**

The student demonstrates competent and proficient performance and shows a thorough and effective application of knowledge and skills that exceeds the standard in **geometry**. The student completes satisfactory tessellations for both labs and successfully completes the first Explore More activity. The student develops well written explanations for most questions on the lab sheets.

- **Mastery**

The student demonstrates fundamental course or grade level knowledge and skills by showing consistent and accurate academic performance that meets the standard in **geometry**. The student completes satisfactory tessellations for both labs, but does not complete any of the Explore More activities. The student develops satisfactory explanations for most questions on the lab sheets.

- **Partial Mastery**

The student demonstrates basic but inconsistent performance of fundamental knowledge and skills characterized by errors and/or omissions in **geometry**. Performance needs further development. The student completes a satisfactory tessellation for Lab 1, but does not complete a satisfactory tessellation for Lab 2. The student develops incomplete or inaccurate explanations for most questions on the lab sheets.

- **Novice**

The student demonstrates substantial need for the development of fundamental knowledge and skills, characterized by fragmented and incomplete performance in **geometry**. Performance needs considerable development. The student is unable to complete satisfactory tessellations for either of the labs. The student develops very sketchy, inaccurate explanations for questions completed on the lab sheets.

Extensions:

1. There is another lab available in Exploring Geometry in which a rotation tessellation is made using an irregular tile created from an equilateral triangle.
2. Students could do research on tessellations or create them using internet searches and interactive websites.

Adaptations and accommodations:

1. Provide oral and written instructions.
2. Have students highlight key points of written materials.
3. Allow students to work in pairs.
4. visual/spatial learner: Provide pictures to show what the student should have on the screen after each step of activity.
5. bodily/kinesthetic learner: Provide pattern blocks or other manipulatives for the students to practice creating tessellations before they complete these labs.
6. interpersonal learner: Provide an opportunity for these students to discuss their results with one or more classmates or to complete the lab with a partner.

Resources used:

1. Seeley, Cathy and Barbara Alcalá. Foundations of Algebra and Geometry. Menlo Park: Addison Wesley Longman, 1998.
2. Bennett, Dan. Exploring Geometry. Emeryville: Key Curriculum Press, 1999.

Websites:

1. www.mathforum.org
This site offers great resources for both teachers and students. A search using the word tessellation provided many other links for lesson plans, information, and student interactive opportunities.
2. www.tessellations.com/
This site offers introductory tessellation lessons for teachers as well as materials for sale related to tessellations.
3. <http://library.thinkquest.org/16661/index2.html>
This site provides students with a lot of introductory information on tessellations.
4. <http://www.shodor.org/interactivate/activities/tessellate/>
This site contains a great interactive tessellation activity for students.

Tessellations

A tessellation is an arrangement of figures that fills a flat surface but does not overlap or leave gaps. Tessellating, or covering a surface with identical figures, is a way to create designs. When the same figure is used throughout, the tessellation is called a *pure tessellation*. You see tessellations on floor covering, wallpaper, fabric, and other decorated objects.

Figures that have exactly the same shape and size are *congruent figures*. Congruent shapes appear in many designs. The basic shape is used over and over, but it is transformed by using translations (slides), reflections (flips), and rotations (turns). A *transformation* is a movement of points that transforms, or changes, a figure. Translations, reflections, and rotations transform figures into congruent figures.

Translating a figure means sliding all points in the figure the same distance in the same direction. The image of a translation is the new figure that results from the slide. The original and its image have the same size and shape, and they face in the same direction. Translating the same figure in the same way again and again makes a design. Artists and creators of handicrafts use these translations to create interesting designs.

If a figure is flipped over a line, the image is a reflection of the original. You get a design that has line symmetry. A reflection transforms points into mirror images. An object and its reflection have the same size and shape, but figures face in the opposite direction.

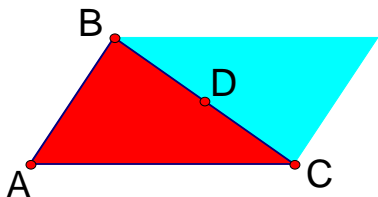
A rotation is a transformation that turns a figure about a fixed point called the *center of rotation*. The angle of turning is called the *angle of rotation*. An object and its rotation have the same size and shape, but are figures in different positions.

Mathematicians use transformations to help them understand nature. Artists use transformations to help them create works of art. Transformations can produce designs that have translational, reflectional, or rotational symmetry.

Seeley, Cathy and Barbara Alcala. Foundations of Algebra and Geometry. Menlo Park: Addison Wesley Longman, 1998.

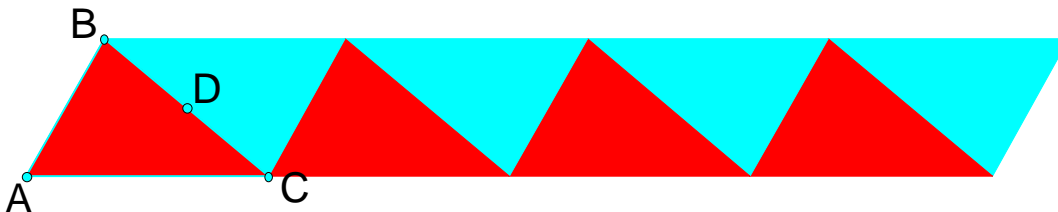
In this lab you will learn a method for tessellating a plane with any kind of triangle. You will also discover the reason why triangles in general tessellate. Use **Geometer's Sketchpad** for this activity.

1. Make sure that under **display** you click *preferences* and have points automatically labeled checked.
2. In the lower left hand corner of your screen, using the segment tool, make a triangle ABC. What kind of triangle did you make? _____



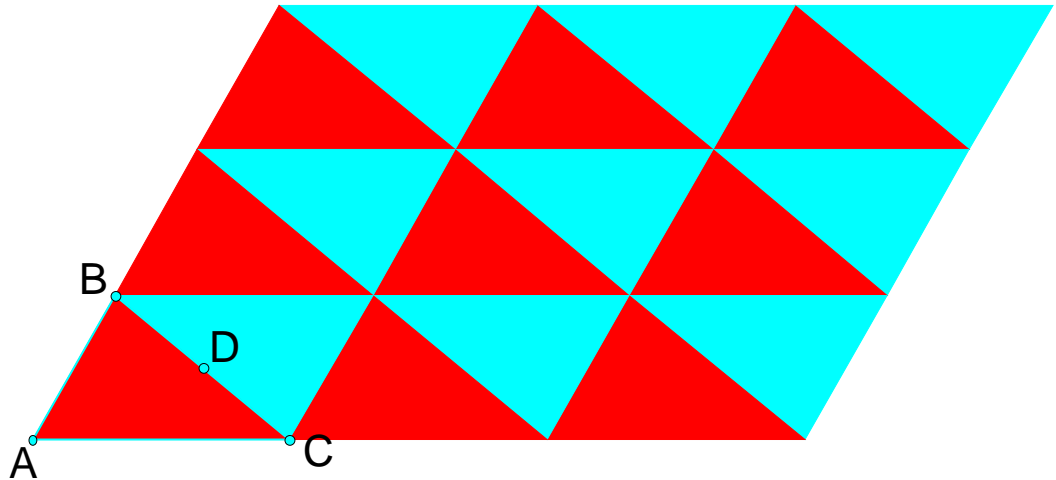
3. Highlight the three vertices of your triangle, then go under **Construct** and choose *Polygon or Triangle Interior*. You may color your interior by choosing a color under **Display**.
4. Select segment BC and under **Construct** choose *midpoint*. This should be labeled D. Select point D and under **Transform** *mark D as a center*.
5. Select the triangle interior and under **Transform** choose *rotate* and rotate the figure 180° . Give the rotated image a different color (under **Display**).
 - Drag points and observe the shape formed by the two triangles (the original together with its rotated image). What shape is this?

6. Mark AC as a vector by selecting, in order, point A then C, then under the **Transform** menu choose *Mark vector*. A brief animation indicates the vector marked. Select the two interiors, then in the **Transform** menu choose *translate*. Repeat this translation as many times as you need to completely go from one edge of the screen past the opposite edge.



- Drag to confirm that the top and bottom edges of this row of triangles are always straight lines. What does that demonstrate about the sum of the three angle measures in the original triangle? Explain this *in sentence form* in the space below.

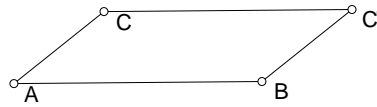
7. Mark vector AB and *translate* the entire row by this vector. Repeat until triangles begin to fill your screen.



8. Drag to confirm that no matter what shape your original triangle has, it will tessellate.
- Look at a point in the tessellation that is completely surrounded by triangles. What is the sum of the angles surrounding this point? Why? Write your explanation *in sentence form* in the space below.
9. Continue in marking vectors so that the entire screen is tessellated by your transformation.
10. Save your tessellation on the disk provided (a:) with **.gsp** on its name.

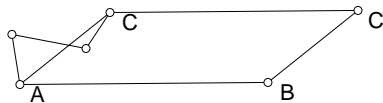
In this lab you will learn how to tessellate a plane using translations with an irregularly shaped tile constructed from a parallelogram base.

1. Under **file**, select *new sketch*.
2. Construct segment AB in the lower left corner of your sketch using the segment tool. Then construct point C just above segment AB using the point tool.
3. Using the select tool, highlight, in order, point A and point B. Remember to hold down the shift key to select more than one item. Go under **Transform** and choose *Mark Vector* to mark the vector from point A to point B. Now select point C and choose *Translate* under **Transform** to translate point C by the marked vector.
4. Use the segment tool to finish constructing a parallelogram



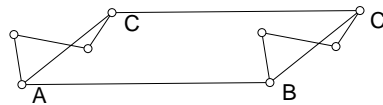
Steps 2-4

5. Form irregular edge AC by constructing two or three connected segments from point A to point C.



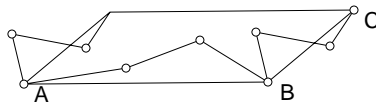
Step 5

6. Holding down the shift key, select the segments and points of irregular edge AC and under **Transform** choose *Translate* to translate irregular edge AC by vector AB. Vector AB should still be marked from step 3 above.



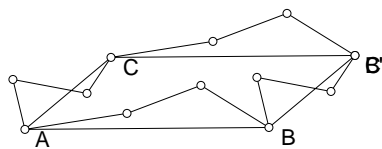
Step 6

7. Construct another irregular edge from A to B.



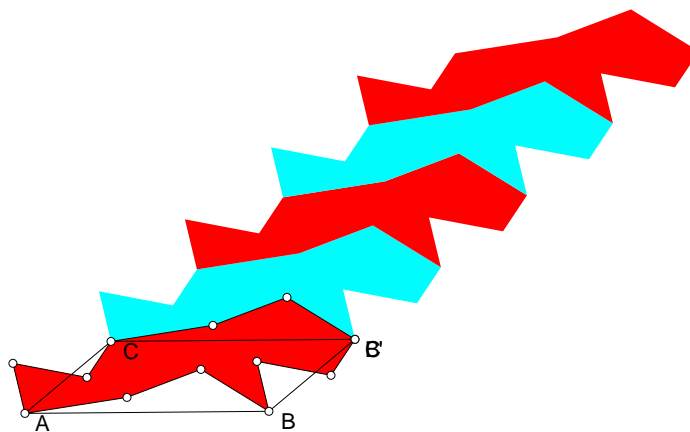
Step 7

8. Mark AC as a vector under **Transform**. Select all points and segments of irregular edge AB and translate them by vector AC using *Translate* under **Transform**.



Step 8

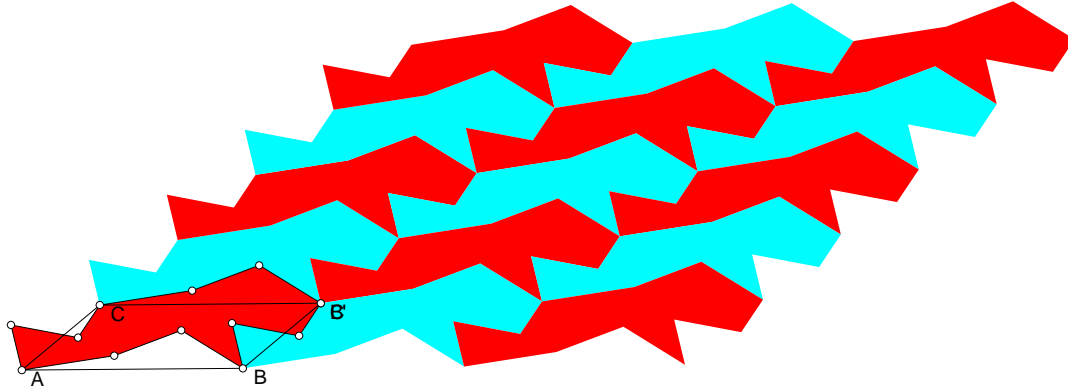
9. You now see the outline of an irregular polygon. Select, in consecutive order, all of the vertices of the irregular polygon and choose *Polygon Interior* under **Construct** to form the interior of the irregular polygon. Choose *color* under **Display** and pick a color of your choice for the polygon interior. You will now tessellate your computer screen by translating with the polygon tile you constructed.
10. Vector AC should still be marked from step 8. Translate the polygon interior by vector AC.
11. Continue translating the polygon interior by vector AC until the height of your computer screen is filled with polygon tiles. Select alternating tiles and choose a second color for them.



Steps 9-11

12. Mark vector AB again. Translate your entire column of polygon tiles by selecting all of the tiles and choosing *Translate* under **Transform**.

13. Fill your computer screen by continuing to *translate* columns of tiles by vector AB. Change colors of alternating tiles to maintain a pattern for the tessellation. Make sure that the screen is completely filled. Mark vectors again or in a different order to fill in all gaps around the screen.



Steps 12 and 13

When you have finished tessellating your entire screen, please save your sketch in directory a: after inserting your floppy disc. Please name your sketch using the extension gsp.

14. What characteristic of triangles and parallelograms allow them to tessellate the plane? What other geometric figures will tessellate the plane? What geometric figures will not tessellate the plane? Use complete sentences for all responses.

Extensions

1. Drag one or more of the original vertices of your irregular polygon until you obtain a figure recognizable to you (examples – animals, people, etc.). Describe the figure you see in three complete sentences. Please save this variation of your original sketch on your floppy disk also. Make sure to give it a new name with .gsp as the extension also.

2. Animate your tessellation by constructing a small circle or short segment near one or two of your original parallelogram points. This circle or short segment will define a path for your animation. Select each point and its nearby path. Under **Edit**, choose *Animation*. Please assign your animated sketch a new name and save it with your other sketches on your floppy disk also.

3. Open a new sketch and tessellate your computer screen using a tile created by beginning with a regular hexagon. (Hint: The process is very similar; it just involves a third pair of sides. Save your sketch on the same floppy disc with your earlier tessellation sketches.

(Lab 1 and Lab 2 were adapted from activities in *Exploring Geometry* by Key Curriculum Press, 1999)