The Hexaflexagon and My Learning Community

by

Ronald E. Schleger

The Hexaflexagon and My Learning Community

by Ronald E. Schleger
Monongalia/Preston/Garrett County School Systems

Content Standards and Objectives:

- 10.3.1
- G.3.9

Materials:

- Construction tools
  - Rulers
  - Compasses
  - Protractors
  - Pencils
- Markers
- Transparent tape
- Scissors

References:

- Tom Repine, Education Specialist, West Virginia Geological and Economic Survey, various geologic hexaflexagons.

Websites (as per Alta Vista, Keyword: hexaflexagon):

http://www.volcanoworld.org/vwdocs/vwlessons/activities/h_number7.html
(This site is a good example of showing island development.)

http://www/enchantedlearning.com/math/geometry/hexaflexagon/print.shtml
(This site has a different perspective. It also talks about the inventor.)

http://www.kathrynhuxtable.com/cgi-bin/home/flexagon/flexer
(This is a good site with interesting variations. It is also interactive.)

This lesson is about creating and using a hexaflexagon in the classroom. It is divided into two distinct sections. One section provides directions for constructing the pattern of a hexaflexagon. Another section provides directions for folding the hexaflexagon. Uses of the hexaflexagon have been supplied by my learning community.

My target group contains students with IEP's but could be adapted to more advanced students simply by requiring a more demanding level of construction techniques. Also, web research can be incorporated. The IEP's that my students had, included:

- needing a calculator during a test
- reading the test to them
- requiring a copy of anything that I put on the board

The student will have satisfactory achievement in constructing patterns.

Procedure: Folding a Hexaflexagon

- Print out and cut the perimeter on the supplied hexaflexagon.

- Folding technique
  - Fold on any side of a rhombus such that the borderline is visible in the folded position. Unfold and
  - Repeat until all sides of all rhombuses have been folded as described above (ten folds total).
  - Fold on the dotted short diagonals such that the dotted line is visible in the folded position. This will establish where the crease should be. Now, reverse the fold and make a sharp crease. Unfold and
  - Repeat until all short diagonals have been folded (six folds total).
- Hint: Sharp creases/folds will make the hexaflexagon work better.

SAFETY NOTE: The type of compasses and scissors used during this lesson should be coincided with the student's abilities.

Teaching Suggestion

- The size of the pattern used has a big effect on how well your hexaflexagon will work. I supplied measurements that will result in an orange-sized unit. I also supplied two patterns that are ready to print as is. One on 8-1/2" x 11" paper and the other on 8-1/2" x 14" paper.

- The weight of the paper used is another important factor on how well your hexaflexagon will work. I found that a heavier weight paper than copy paper but not as heavy as card stock works best for me.

- Precision construction and sharp folds/creases are of utmost importance.

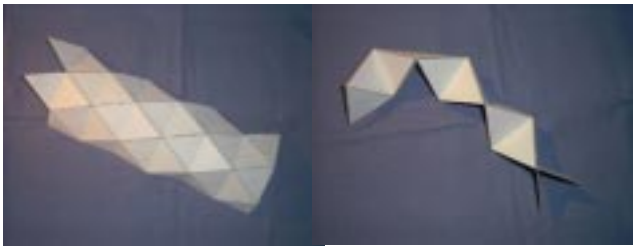
- A pattern, picture, or rough sketch should be posted on the board as a visual aid.

Suggestions for use provided by my Learning Community. Use as:

- An alternative to flashcards.
- A review tool for vocabulary and formulas.
- An aid with triangle and rhombus concepts.
- A fun activity during days before holidays.
- An aid for discussing parallel lines, transversals, angle sum, etc.
- An aid to fully incorporate CSO's of construction and/or investigating measured angles.
- As a review or study guide. For example, put the questions on one face and them turn to the answers on the next face.
- A beginning of the year activity, where personal information is put on each face.
- A enrichment activity--creating patterns using parallel lines and transversals.

- Assembly technique
  – Bring the "A" section to fit over triangle A', align and tape.
  – Bring the "B" section to fit over triangle B', align and tape.
  – Bring the "C" section to fit over triangle C', align and tape the upper ½ of this section only.
  – Leave the rhombus corners labeled "D" and "E" untaped.
  – Bring triangle D' and align over triangle F' and tape it.
  – Bring rhombus corner "D" over triangle D', align and tape it.
  – Bring rhombus corner "E" over triangle E', align and tape it.
  – Turn hexaflexagon and check for open edges, tape as needed.

- Hints:
  – Sharp creases/folds will make the hexaflexagon work better.
  – The faint letters "A," "B," "C," "D," and "E" should be visible when done.
  – Note that a 6" x 15" pattern will result in a hexaflexagon of about orange size.
  – On a completed hexaflexagon lightly draw/sketch in pencil the appropriate design on a 3- rhombus-face, repeat three more times. When satisfied with results darken with bold markers.



Adaptations

- For students other than the ones described, they can be accommodated by requiring a more demanding level of construction techniques. Also, web research can be incorporated.
- Have the hexaflexagons already assembled for certain students.
- Information can be put on the patterns prior to assembly.

Learning Styles Accommodated

- Verbal/Linguistic
- Logical/Mathematical
- Visual/Spatial
- Bodily/Kinesthetic

Questions for the Students

What would happen if the angles were changed on the hexaflexagon?

What changes would occur to the shape if more rhombuses were added?

Procedure: Constructing a Large Hexaflexagon

- Display a rough sketch of a hexaflexagon on the blackboard.
- Construct in landscape orientation a rectangle with sides of 13-5/8" x 5-3/8".
- On the top base, mark the following increments from left to right: 1-15/16", 5-13/16", 9-11/16", 11-5/8", and 13-5/8".
- On the bottom base, mark the following increments from left to right: 3-7/8", 7-3/4", 11-11/16" and 13-5/8".
- On left side, mark from top to bottom the following increments: 1-1/16", 3-3/16", 5-5/16".
- On right side, mark from top to bottom the following increments: 2-3/16", 4-5/16".
- Using the posted hexaflexagon as an example, construct rhombuses connecting the appropriate points.
- Draw dotted lines as short diagonals on all rhombuses.
- Layout the perimeter as displayed on blackboard and cut away excess.

- Hint: All labels should be on outside of construction.

Assessment:

- The student will be assessed on the correctness of the physical hexaflexagon.

- The student will be assessed by the information that is printed/drawn on the finished product.

Further Challenges:

- The student could do web research.
- The student could modify the hexaflexagon's dimensions, such as the length of the short diagonals or the total number of rhombuses in the length.
- The student could orient the information on the pattern prior to assembly.

OBJECTIVES — Content Standard 3:

10.3.1 Use appropriate tools to make geometric constructions. (AGP.16)

Performance Descriptors:	
Distinguished	**The student demonstrates exceptional and exemplary performance with distinctive and sophisticated application of knowledge and skills that exceeds the standard in geometry. A student uses appropriate tools to make geometric constructions and justifies the results.** The student solves, interprets and defends the reasonableness of solutions in a clear concise manner to multi-step practical application problems involving angle relationships, relationships formed by parallel lines cut by a transversal, similar figures, circle relationships and the Pythagorean Theorem.
Above Mastery	**The student demonstrates competent and proficient performance and shows a thorough and effective application of knowledge and skills that exceeds the standard in geometry. A student uses appropriate tools to make geometric constructions** and solves practical application problems giving solutions in a clear manner. The student uses the parts of a circle and their relationships to solve practical application problems giving solutions in a clear manner. The student solves practical application problems using the Pythagorean Theorem giving solutions in a clear manner.
Mastery	**The student demonstrates fundamental course or grade level knowledge and skills by showing consistent and accurate academic performance that meets the standard in geometry. A student uses appropriate tools to make simple geometric constructions** and solves simple problems involving angle relationships (such as complementary, supplementary, vertical and adjacent angles) as well as relationships formed by parallel lines cut by a transversal. The student identifies similar figures and uses proportions to solve simple problems and identifies the parts of a circle and their relationships. The student can find a missing side in right triangle problems using the Pythagorean Theorem.
Partial Mastery	**The student demonstrates basic but inconsistent performance of fundamental knowledge and skills characterized by errors and/or omissions in geometry. Performance needs further development. A student uses appropriate tools to copy a segment**, bisect an angle and perpendicularly bisect a segment and identifies basic angle pairs such as complementary, supplementary, vertical and adjacent angles. The student identifies similar figures and parts of a circle and can use Pythagorean Theorem in simple problems.
Novice	**The student demonstrates substantial need for the development of fundamental knowledge and skills, characterized by fragmented and incomplete performance in geometry. Performance needs considerable development**. A student identifies the parts of a circle and basic angles pairs such as complementary, supplementary, vertical and adjacent angles.

Content Standards and Objectives (MA.S.3)

G.3.9 Investigate measures of angles and lengths of segments to determine the existence of triangles (triangle inequality) and the order of sides and unknown side lengths or angles and inaccessible heights and distances, construct scaled drawings, and derive the basis for the trigonometric ratios.

Performance Descriptors (MA.PD.G.3)

Distinguished

The student demonstrates exceptional and exemplary performance with distinctive and sophisticated application of knowledge and skills that exceeds the standard in geometry. The student uses the converse, inverse, and contrapositive of a conditional statement and Venn diagrams to develop and test arguments. **The student applies definitions, theorems and postulates to explore angles formed by lines, including parallel and perpendicular**, and determines measures of angles found in figures containing polygons and circles **and justifies solutions in a clear, concise manner**. Students apply appropriate formulas to solve complex practical application problems involving area, perimeter, surface area, and volume expressing solutions in both exact and approximate forms in a clear, concise manner. The student develops proofs and differentiates among inductive, deductive, direct and indirect methods included in both formal and informal logical arguments. The student solves complex application problems involving triangles by applying properties of congruence, similarity, the Pythagorean Theorem, special right triangles, and right triangle trigonometric ratios justifying conclusions in a clear concise manner. The student compares and contrasts other geometries to Euclidean geometry.

Above Mastery

The student demonstrates competent and proficient performance and shows a thorough and effective application of knowledge and skills that exceeds the standard in geometry. The student verifies properties of geometric figures and polygons. The student explains the converse, inverse, and contrapositive of a conditional statement and uses Venn diagrams to test the validity of arguments. **The student applies definitons, theorems, and postulates to determine measures of angles formed by lines, including parallel and perpendicular**, as well as angles found in figures containing polygons and circles. Students apply appropriate formulas to solve practical application problems involving area, perimeter, surface area, and volume. The student recognizes examples of inductive, deductive, direct and indirect proofs that include both formal and informal logical arguments. The student solves practical triangle application problems by applying properties of congruence, similarity, the Pythagorean Theorem, special right triangles, and right triangle trigonometric ratios. The student applies transformations and uses analytical geometry to explain practical mathematical situations and recognize the existence of non-Euclidean geometries.

Mastery

The student demonstrates fundamental course of grade level knowledge and skills by showing consistent and accurate academic performance that meets the standard in geometry. The student identifies and represents basic geometric figures such as points, lines, planes and polygons and uses basic properties of each to solve problems. The student recognizes the converse, inverse, and contrapositive of a conditional statement and constructs Venn diagrams depicting intersections and unions. **The student finds measures of angles found in figures containing polygons** and circles **as well as those formed by lines, including parallel and perpendicular, using definitions, theorems and postulates**. The student solves problems involving area, perimeter, surface area, and volume by using appropriate formulas. The student solves triangle problems involving congruence, similarity, and the Pythagorean Theorem and creates basic transformations. The student uses coordinate geometry to determine distance, slope and midpoint.

Partial Mastery

The student demonstrates basic but inconsistent performance of fundamental knowledge and skills characterized by errors and/or omissions in geometry. Performance needs further development. The student identifies basic geometric figures such as points, lines, planes, triangles and quadrilaterals and inconsistently uses basic properties of each to solve problems. Given definitions, theorems, and postulates, **the student inconsistently finds measures of angles found in figures containing triangles, quadrilaterals and regular polygons as well as those formed by lines**. Given appropriate formulas and detailed drawings, the student inconsistently solves problems involving area, perimeter, surface area, and volume. Given the properties, a student inconsistently solves simple triangle problems involving congruence, similarity, and the Pythagorean Theorem.

Novice

The student demonstrates substantial need for the development of fundamental knowledge and skills, characterized by fragmented and incomplete performance in geometry. Performance needs considerable development. The student recognizes some of the basic geometric figures such as points, lines, triangles, quadrilaterals and basic angle pairs. Given appropriate formulas and tools, a student attempts to determine the area and perimeter of triangles and quadrilaterals.

Suggestions from the Learning Community on:

Procedures

Teaching Suggestions

Adaptations

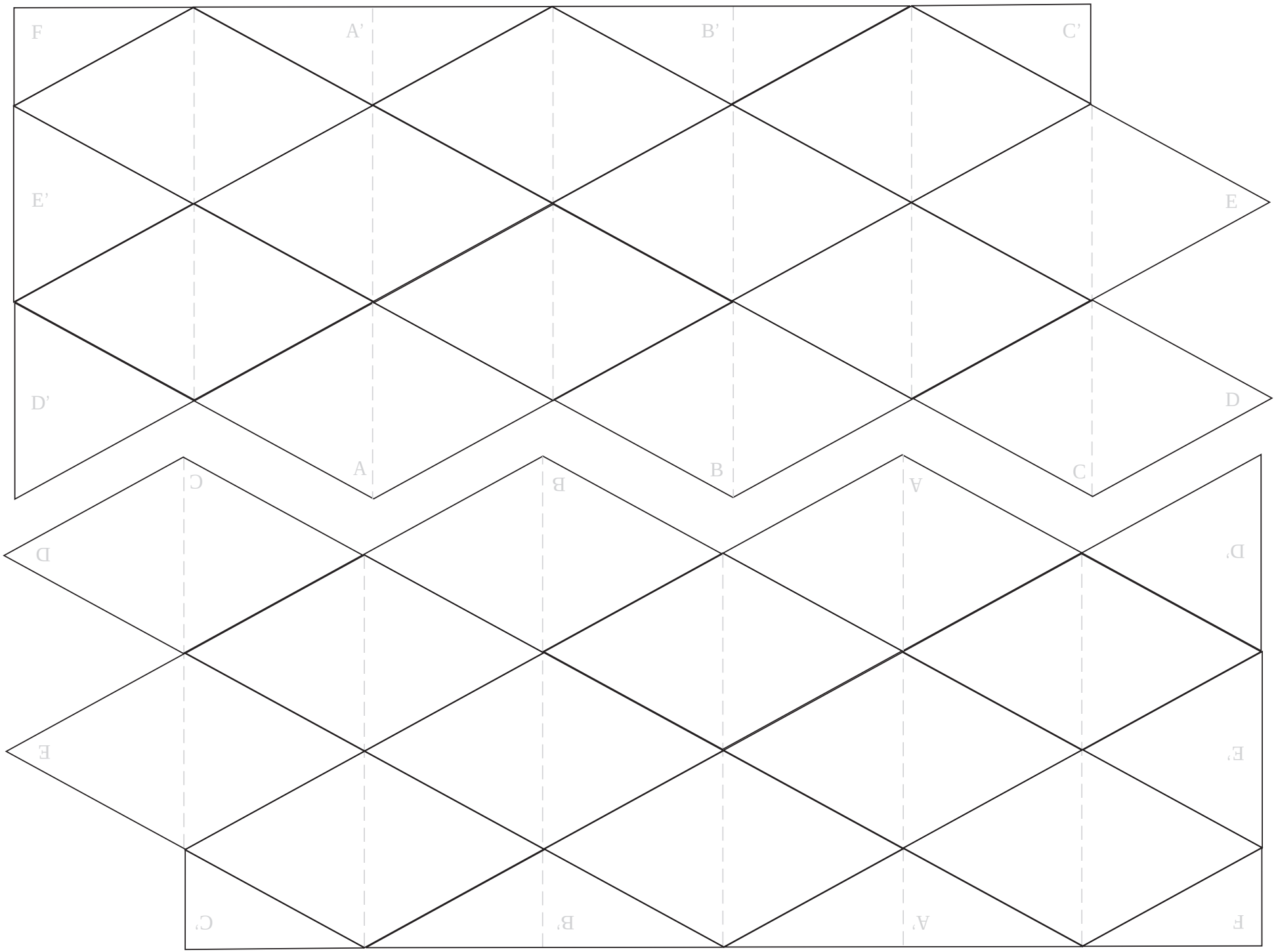
Assessment

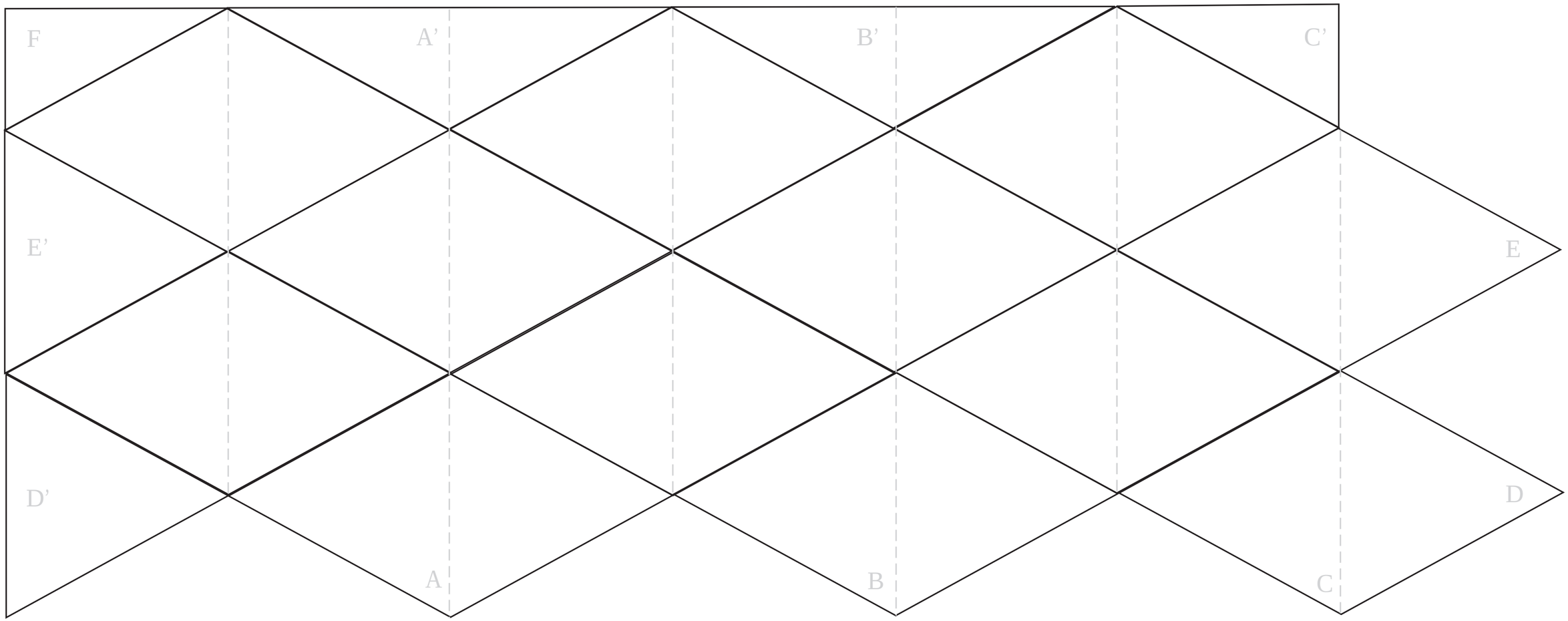
Further Challenges

Questions

How would you use it?

Other


F
A’
B’
C’
E’
E
D’
D
A
B
C
C
B
A
D
D’
E
E’
C’
B’
A’
F


F
A’
B’
C’
E’
E
D’
D
A
B
C