Hoop Shooting and Other Sweet Ideas

Blue Ribbon Probability Workshop
Middle School/High School Activities

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Hoop Shooting and Other Sweet Ideas

Introduction:

Students will use simulations and activity sheets to learn about situations that are modeled by binomial experiments. The Candy Family activity has been adapted to two levels – middle school/applied mathematics and high school. At the middle school/applied math level, the student utilizes basic probabilities and tree diagrams along with an M&M simulation to explore the possible arrangements and probability distributions for families of various sizes. Sherea added student graphing the results of the M&M simulation as part of their discussion. At the high school level, the student transitions from tree diagrams to binomial probabilities. The Hoop Shooting activity is appropriate only for the high school level. It would be too tedious to do with a tree diagram, and requires not only the use of binomial distributions but also knowledge of combinations to create the coefficients in the distribution. Students indicated that they enjoyed both activities.

Goals:

- Explore binomial distribution problems
- Approach binomial probability problems through simulation of binomial experiments
- Use binomial distribution capabilities of the graphing calculator to confirm calculations of theoretical probabilities for binomial distribution problems

West Virginia Content Standards:
Standard 5: Data Analysis and Probability (MA.S.5)
Students will:

- Apply and demonstrate an understanding of basic concepts of probability through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics.

Objectives:
Students will:

MA.7.5.1	determine experimental and theoretical probability of an event using appropriate technology
MA.7.5.2	construct sample spaces by listing, tree diagrams, and frequency distribution tables to determine combinations and permutations.
MA.8.5.2	investigate the experimental and theoretical probability, including compound probability of an event.
MA.8.5.4	analyze problem situations, games of chance, and consumer applications using statistical samplings to determine probability and make predictions.
AGP.5.1	read, interpret and construct graphs to solve problems

AGP.5.3 find the probability of complementary events and exclusive events.
A1.2.20 predict the outcomes of simple events using the rules of probability.
AM1.2.16 predict the outcomes of simple events using the rules of probability
CM.5.2 integrate other disciplines into the study of mathematics through simulations, research, and projects
CM.5.3 determine possible outcomes using tree diagrams and the counting principles of permutations and combinations.
CM.5.4 apply the basic probability rules in expressing the chances of events occurring, using technology when appropriate.
CM.5.5 create and interpret data using various methods of displaying numerical data, including frequency distributions, graphs histograms, stem-and-leaf plots, and box-and –whiskers plots, sing technology when appropriate.
PS.5.1 distinguish between experimental and theoretical probability.
PS.5.3 determine possible outcomes using tree diagrams and the counting principles of permutations and combinations.
PS.5.4 express the chances of events occurring either in terms of a probability or odds.
PS.5.5 use the normal distribution and the binomial distribution including Pascal's triangle, to determine probability of events.

Materials and Equipment for "The Candy Family"

4 M&M's and 5 sticky notes in a baggie for each student
A Fun Size bag of M&M's to give each student upon completion of the Activity – or they will eat the 4 dirty M&M's
Activity pages
Calculator

Materials and Equipment for "Hoop Shooting"

Basketball hoop
Basketball
Activity sheets
Calculator

Time Required
One ninety minute block period for each of the two activities.

Prior Knowledge

At all levels it is helpful for students to know how to predict the outcomes of simple events using the rules of probability. For "Hoop Shooting", it saves time when computing the coefficients of the binomial distribution if students are familiar with combinations.

Evaluation:

Students will receive a grade based upon their completed activity sheets. The grade will be referenced to performance descriptors developed for the labs.

Performance Descriptors for Middle/Applied Mathematics:

- **Distinguished**
  The student demonstrates exceptional and exemplary performance with distinctive and sophisticated application of knowledge and skills that exceeds the standard in data analysis and probability. ***The student conducts the "Candy Family" experiment and uses the information to give well written explanations for all questions on the activity sheets. The student will use the obtained data to accurately construct a histogram. The student will also complete the extension activity.***
- **Above Mastery**
  The student demonstrates competent and proficient performance and shows a thorough and effective application of knowledge and skills that exceeds the standards in data analysis and probability. ***The student conducts the "Candy Family" experiment and uses the information to give well written explanations for all questions on the activity sheets. The student will use the obtained data to accurately construct a histogram.***
- **Mastery**
  The student demonstrates fundamental course or grade level knowledge and skills by showing consistent and accurate academic performance that meets the standard in data analysis and probability. ***The student conducts the "Candy Family" experiment and uses the information to give satisfactory answers for most questions on the activity sheets. The student will use the obtained data to accurately construct a histogram.***
- **Partial Mastery**
  The student demonstrates basic but inconsistent performance of fundamental knowledge and skills characterized by errors and/or omissions in data analysis and probability. Performance needs further development. ***The student conducts the "Candy Family" experiment and uses the information to answer the basic questions and probabilities.***
- **Novice**
  The student demonstrates substantial need for the development of fundamental knowledge and skills, characterized by fragmented and incomplete performance in data analysis and probability. Performance needs considerable development in using appropriate statistical methods.

Performance Descriptors for High School:

- **Distinguished**
  The student demonstrates exceptional and exemplary performance with distinctive and sophisticated application of knowledge and skills that exceeds the standard in probability and statistics. The student completes both activities and goes beyond by successfully completing the extended activities without help from the instructor. The student develops outstanding written explanations for all questions on the activity sheets.
- **Above Mastery**
  The student demonstrates competent and proficient performance and shows a thorough and effective application of knowledge and skills that exceeds the standard in probability and statistics. The student completes both activities and goes beyond by successfully completing the extended activities with minimal help from the instructor. The student develops well-written explanations for most questions on the activity sheets.
- **Mastery**
  The student demonstrates fundamental course or grade level knowledge and skills by showing consistent and accurate academic performance that meets the standard in probability and statistics. The student completes satisfactory activity sheets for both activities, but does not complete the extended activities without considerable help from the instructor. The student develops satisfactory explanations for most questions on the activity sheets.
- **Partial Mastery**
  The student demonstrates basic but inconsistent performance of fundamental knowledge and skills in probability and statistics characterized by errors and/or omissions. The student completes satisfactory work in the first activity with basic probabilities and tree diagrams, but has incomplete or inaccurate work on the binomial probabilities. The student does not answer questions with well-worded sentences. The student does not complete the extensions.
- **Novice**
  The student demonstrates substantial need for the development of fundamental knowledge and skills, characterized by fragmented and incomplete performance in probability and statistics. The student is unable to complete satisfactory work in either activity and does not complete either extension. Explanations are brief and sometimes inaccurate.

Websites:
http://faculty.vassar.edu/lowry/binomialX.html
This page will calculate and/or estimate binomial probabilities for situations of the general "k out of n" type.

http://espse.ed.psu.edu/edpsych/faculty/rhale/statistics/Chapters/Chapter7/Chap7.html#Binomial%20Probability
This site is an excellent source of information about different types of probability.

http://www.richland.edu/james/lecture/m170/ch06-bin.html
This site offers an in-depth explanation of binomial probability as well as examples and counter examples to illustrate.

http://www-stat.stanford.edu/~naras/jsm/example5.html
This site has an applet that will generate a histogram based on the number of trials and the probability of success.

https://www.nsa.gov/teachers/es/prob64.pdf
This site provides an excellent set of basic beginning probability activities for lower level students

http://archon.educ.kent.edu/Oasis/Resc/Educ/data.html
This site provides some activities that start basic (and fun). Then it has a link to real-life (mortality rates) to peak interest for older students.

Sources:
The activities were designed by Debby to be used in her Probability/Statistics classes. Resources included:

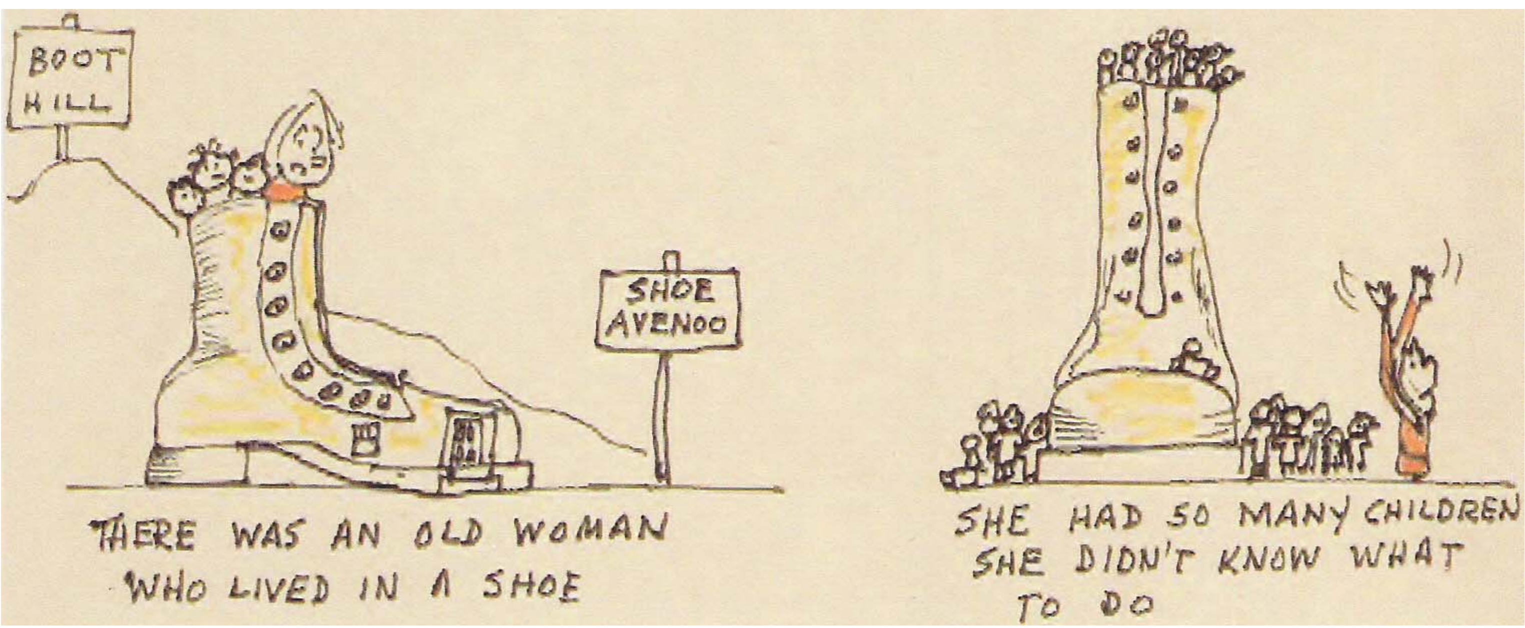
Vertical Teams Pre-AP Consultant Materials: Setting the Cornerstones in Mathematics

Navigating through Probability in Grades 9-12, National Council of Teachers of Mathematics, 2004

Who Did What?

Debby wrote the activities and power point for Candy Family and Hoop Shooting. She also wrote the portion of the plan for high school. Sherea adapted the high school version to be appropriate for applied mathematics students. Jared adapted the activities for use at the middle school. Sherea and Debby did the staff development with PHS teachers. Debby, Sherea, and Jared used the activities in the classroom.

The
Candy Family



The Candy Family

1. If a family decides to have 4 children, how many boys and how many girls will they have?

2. We will use a simulation to discover the distribution of boys and girls.

   a. Select 4 pieces of candy. Place them in a cup. Shake the cup and roll the candy on the table. An M up is a boy, no M up is a girl.
   b. Write the number of boys and girls on a sticky note.
   c. Repeat steps a and b several times.
   d. Form a histogram using the sticky notes.

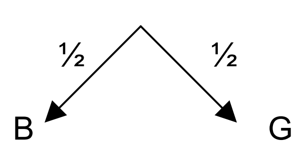
3. Based on your simulation, what is the most likely distribution of boys and girls? Least likely distribution?

4. In this family of 4 children, what is the probability of having 3 boys and 1 girl?

   To solve this problem, it may help to look at a simpler problem.

   If a family decided to have one child, what is the probability that the child if a boy? ______________ a girl? ________________

   Probabilities such as this are sometimes represented with a tree diagram:



P(B) = __________ P(G) = _______________

What is the sum of the probabilities in this distribution? P(B) + P(G) = ______
The Candy Family, Page 2

But what if the family decides to have two children?

What arrangements of children are possible in this case?

_____________ _______________ _______________ _______________

Make a tree diagram to illustrate the two children family distribution.

If this information is organized, it is possible to see a pattern that will help predict family distributions for any size family.

P(BB) = ________ P(BG) = ________ P(GB) = ________ P(GG) = ________

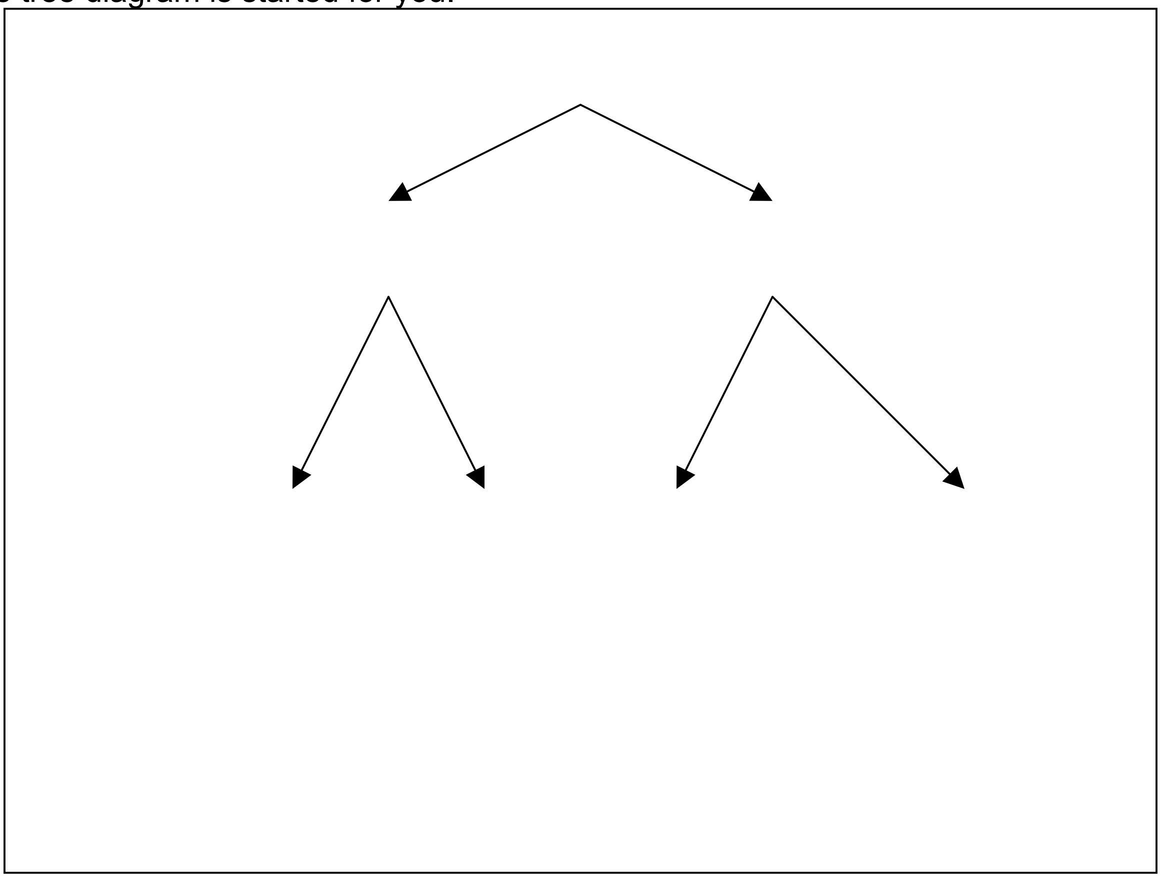
P(2B) = P(1B1G) = P(2G) =

As in the case of a family of one child, the sum of the probabilities in this distribution of a 2 child family distribution is also ___________.

The Candy Family, Page 3

If we continue this process, how many branches will our tree diagram need to determine the probabilities for a family of 4 children? ____________________

Construct a tree diagram for a family of 4 children. Be sure to label the probability on each branch and B for boy and G for girl at the end of each branch. The tree diagram is started for you.



How many possible arrangements for boys and girls in your completed tree diagram? ____________________

Name them. Be sure to follow all possible branches.

__________ __________ __________ __________ __________

__________ __________ __________ __________ __________

__________ __________ __________ __________ __________

____

The Candy Family, Page 4

What is the probability of each one of the above arrangements? ______________

How many arrangements result in 4B0G? _____________ P(4B0G) = _________

3B1G? _____________ P(3B1G) = _________

2B2G? _____________ P(2B2G) = _________

1B3G? _____________ P(1B3G) = _________

0B4G? _____________ P(OB4G) = ________

As before, what is the sum of the probabilities for a 4 child family? ____________

Now! What was the question we were trying to answer? ____________________

____

Write your answer to the question as a sentence.

____

____

How did your theoretical answer compare to the graph that you constructed with sticky notes?

The Candy Family, Page 5
Extension:

How many children are in your family? ___________________

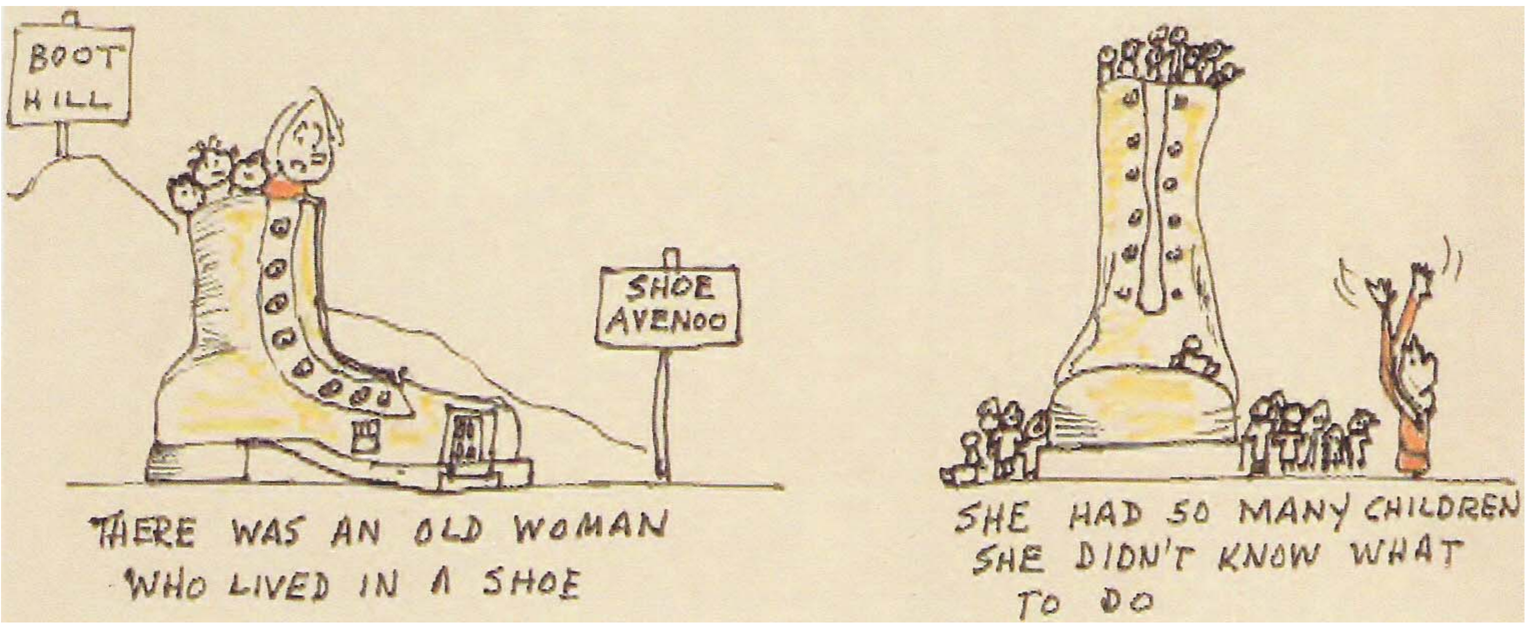
How many boys?_____________ How many girls?___________

Construct a tree diagram in the box provided to represent all possible arrangements of the number of children in your family?

What is the probability for your family's distribution?_________________

State in words how you arrived at this probability?

The
Candy Family



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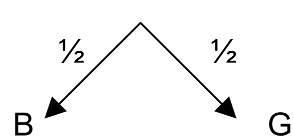

P(B) = __________ P(G) = _______________

What is the sum of the probabilities in this distribution? P(B) + P(G) = ______

The Candy Family, Page 2

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If this information is organized, it is possible to see a pattern that will help predict family distributions for any size family.

P(BB) = ________ P(BG) = ________ P(GB) = ________ P(GG) = ________

$(1)P(B)^2 P(G)^0$ $(2)P(B)^1 P(G)^1$ $(1)P(B)^0 P(G)^2$

Compute the above probabilities.

As in the case of a family of one child, the sum of the probabilities in this distribution of a 2 child family distribution is also ___________.

We could continue this process of constructing tree diagrams or we could look for a mathematical formula to solve our problem. Fortunately, mathematicians have taken care of this problem for us. Probability questions with these characteristics are called **binomial probabilities.**

The scenario of the Candy Family has four important characteristics:

1. It involves a sequence of one or more repeated trials
2. Each trial has two (and only two) possible outcomes
3. The trials are independent of one another
4. The probabilities of the two outcomes are the same in every trial

In general, a binomial probability problem has probabilities p and q for the two outcomes, where p + q = 1, and if X is the random variable that takes on values 0, 1, 2, …, n, for the number of "successes" in n trials, then the probability distribution for X can be found as follows:

For $0 \le r \le n$,

P(X=r) = $C(n,r) \cdot p^r \cdot q^{(n-r)}$, where $C(n,r) = \dfrac{n!}{(n-r)!r!}$

The coefficient C(n,r) can also be found by looking at the entries in the nth row of Pascal's triangle.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1 etc.

So, now we can finally answer our question.

In this family of 4 children, what is the probability of having 3 boys and 1 girl?

Extension:

What is the probability in a family of 9 children, to have enough boys for the family to have their own team of 9? You may need your calculator for this question. Show your work in the box below.

Sources:

Vertical Teams Pre-AP Consultant Materials: Setting the Cornerstones in Mathematics

Navigating through Probability in Grades 9-12, National Council of Teachers of Mathematics, 2004

Hoop Shooting

Binomial Probability Simulation:

Attempt 10 free throw shots. If you make all ten (or zero) baskets, continue to shoot until you miss (or make) one. You will use this information to calculate the experimental probability of your success on any given shot.

Number of successes = ___________________

Number of misses = ___________________

Total number of shots = _________________

Probability of making the basket = P(B) = _______________

Probability of missing the shot = P(M) = _______________

Explain how this simulation satisfies the four criteria of a binomial probability situation:

1.

2.

3.

4.

Set up and calculate to four decimal places the 11 theoretical probabilities for making each of zero to ten baskets based upon **your** experimental probability.

P(10 Baskets) =

P(9 Basket) =

P(8 Baskets) =

P(7 Baskets) =

P(6 Baskets) =

P(5 Baskets) =

P(4 Baskets) =

P(3 Baskets) =

P(2 Baskets) =

P(1 Baskets) =

P(0 Baskets) =

What is the sum of these probabilities? ____________________

Using the information above, what is the probability that you will make exactly five shots.

…at most five shots.

…more than five shots.

Graphing Calculator Extension:

Binomial probabilities are very easy to compute on a TI Graphing Calculator.

1. To calculate the probability of making r of n shots given P(B), use *binompdf(n,P(B),r).* Binompdf is located under the DISTR menu.

   For example, to calculate the probability of making 5 of 10 shots given P(B)=.7, you will use *binompdf(10,.7,5).* Ans. 0.1029

   Use your calculator to check the 11 binomial probabilities that you computed above. How did you do?

2. To calculate the probability of making *at most* r of n shots given P(B), use *binomcdf(n,P(B),r).* Binomcdf is located under the DISTR menu.

   For example, to calculate the probability of making at most 5 of 10 shots given P(B)=.7, you will use *binomcdf(10,.7,5).* Ans. 0.1503

Now you try it:
Set up a calculator statement to find the probability of making at most five shots for your shooting percentage.

____

What was your answer? _________________________________

3. To calculate the probability of making more than r shots given P(B), use 1-binomcdf(n, P(B), r). Binomcdf is located under the DISTR menu.

   For example, to calculate the probability of making more than 5 shots given n=10, P(B) = .7, you will use 1 – binomcdf(10,.7,5).

   Now you try it:
   Set up a calculator statement to find the probability of making more than five shots for your shooting percentage.

   ____

   What was your answer?

   ____

   Did your answer check with what you did at the top of page 3?

____

Cool, huh!