Blue Ribbon Probability Workshop

August 2006 – April 2007

Lesson Plan: Tree Diagrams and Probability

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While we all shared equally in developing the concepts for this lesson plan, each member was responsible for different components of the lesson.

Donna Bindernagel – Directions to the Race Car Game and the game board, gathering information and resources for the lesson, shared in developing the Optional Day 2 Activity

Carrie Breakey – Discussion Question Handout, writing the lesson outline, gathering information and resources for the lesson, shared in developing the Optional Day 2 Activity

Linda Jackson – TI83 Calculator Directions, Tree Diagram Handout, writing the accommodations, and gathering information and resources for the lesson, shared in developing the Optional Day 2 Activity

Tree Diagrams and Probability

Abstract

This lesson is designed to develop students ability to create tree diagrams and figure probabilities of events based on those diagrams.

Objectives

Upon completion of this lesson, students will:

- be able to create tree diagrams
- be able to figure probabilities based on tree diagrams
- practice adding and multiplying fractions
- be able to explain complementary probabilities

Standards

MA.7.5.2 construct sample spaces by listing, tree diagrams, and frequency distribution tables to determine combinations and permutations.

MA.8.5.2 investigate the experimental and theoretical probability, including compound probability of an event.

MA.8.5.5 draw inferences and construct convincing arguments, including misuses of statistical or numeric information, based on data analysis.

Student Prerequisites

- Arithmetic: Students must be able to:
  - add and multiply fractions.
  - Familiarity with probability
- Technological: Students must be able to:
  - perform basic mouse manipulations such as point, click and drag.
  - use a browser, such as Netscape, for experimenting with the activities. (Optional Day 2 Activity)

Teacher Preparation

Teacher will need:

- Game Board Handout
- Tree Diagram Handout
- Discussion Questions Handout
- Game pieces for each group
- Dice and/or graphing calculators with probability simulators for each group

Students will need:

- Access to a browser (Optional Day 2 Activity)
- pencil and paper

Key Terms

This lesson introduces students to the following terms through the included discussions:

- tree diagram
- experimental probability
- theoretical probability

Lesson Outline

1. Focus and Review

   Remind students what has been learned in previous lessons such as concepts of probability. Possibly use the example of rolling a die and the chances of that die rolling a specific number is 1/6.

   Ask students what the probability is of rolling a 1 or a 2.

   Mention the difference between the experimental probability and the theoretical probability.

   Experimental probability is found by repeating an experiment several times and recording the results.

   Theoretical probability is a number from 0 to 1 that tells how likely something is to happen.

   You can even have the students roll dice themselves, collect data for the class and figure the experimental probability for the class.

2. Objectives

   Let the students know what it is they will be doing and learning today. Say something like this:

   i. Today, class, we will be talking more about probability and how to determine the probability of multiple events, known as compound events. We will learn how to create tree diagrams to determine the probabilities related to compound events.
   ii. Optional Day 2 Activity: Use the computers to simulate compound probability with a Racing Game (http://www.shodor.org/interactivate/lessons/tree.html).

3. Teacher Input

   - Lead the students in a short discussion on trees as data structures.

     A tree diagram can be used to show all the possible outcomes of an experiment.

   - Demonstrate the Race Game with the overhead projector. Place a transparency copy of the game board handout on the overhead. Use to different shaped objects or different color see through objects as game pieces. Play the Race Game with the teacher versus the entire class. Explain the directions. When flipping a coin if heads appears the teacher will move one space, if tails appears the class will move one space on the game board. The first car to reach the finish line wins.

   - Allow the students to break into groups of two and perform this activity. Ask students to perform the activity three times and record their results in the chart on the Results Handout.

   - After students have completed the activity complete the Tree Diagram Handout as a class. Lead the students in a discussion about the possible outcomes.

   - Students will complete the Discussion Questions Handout with remaining time in class. Finish the handout for homework.

4. Closure

   You may wish to bring the class back together for a discussion and verification of their findings. Once the students have been allowed to share what they found, summarize the results of the lesson.

Optional Day 2 Activity

Website: http://www.shodor.org/interactivate/lessons/tree.html

- Have students access the above website.
- Explain to the students how to do the assignment. You should model or demonstrate it for the students, especially if they are not familiar with how to use our computer applets.
- Open your browser to Racing Game with One Die in order to demonstrate this activity to the students.
- Demonstrate some of the functionality yourself such as changing the length of the race, changing the game to an unfair race, and demonstrating how the multiple run panel works.

- Have students begin racing on their individual computers. Make the race an unfair race by making the blue car move on rolls of 1 and 2 and the red car move on rolls of 3, 4, 5, and 6.
- Draw a tree diagram for an unfair race. Mention that the sum of the end probabilities always equal one, which makes them complementary probabilities. Discuss why this must be so.
- Ask the students to change the number of runs to 50,000 in the multiple-run pane. Have students run this configuration 5 or 6 times. Ask them to develop a hypothesis as to what the theoretical probability of an unfair race is based on the experimental data using the applet.
- Independent Practice
    - Have the students create a tree diagram for an unfair two-step race to determine the theoretical probability.
    - Have them show, based on their diagrams, the sum of the final probabilities equal one demonstrating they are complementary probabilities.

Accommodations:

The learning styles that are accommodated by this activity include:

1. Verbal/linguistic – oral as well as written instructions
2. Logical/mathematical – numbers and percentages/probabilities
3. Visual/spatial – pictures and images as well as diagrams
4. Bodily/kinesthetic – hands on activity/movement of the game pieces
5. Interpersonal learners – working in groups with others and discussing the results

Assessment:

Students grade will be determined based on the completion of the worksheets: Results Handout, Tree Diagram Handout, and Discussion Question Handout. In addition, class participation during the Race Game activity will also be considered in calculating students' grades.

Performance Descriptors:

- Distinguished: A student at this level has demonstrated exceptional and exemplary performance. The work shows a distinctive and sophisticated application of knowledge and skills that go beyond course or grade level expectations.
- Above Mastery: A student at this level has demonstrated complete and proficient performance and exceeds the standard. The work shows a thorough and effective application of knowledge and skills.
- Mastery: A student at this level demonstrated fundamental knowledge and skills that meet the standard. The work is accurate, complete and fulfills all requirements. The work shows solid academic performance at the course or grade level.

- Partial Mastery: A student at this level has partially demonstrated fundamental knowledge and skills toward meeting the standard. The work shows basic but inconsistent application of knowledge and skills characterized by errors and/or omissions. Performance needs further development.
- Novice: A student at this level has not demonstrated the fundamental knowledge and skills needed to meet the standard. Performance at this level is fragmented and/or incomplete and needs considerable development.

Websites:

- http://www.shodor.org/interactivate/lessons/tree.html
- http://nces.gov/nceskids/createagraph/index.asp
- http://www.mathsonline.co.uk/nonmembers/resource/plans/tree_plan.html
- http://regentsprep.org/Regents/math/tree/<tree.htm

References:

- Billstein, Rick, and Jim Williamson. MathThematics Book 3. Evanston, IL: McDougal Littell, 2005

Race Car Game Directions

Set-up: To set up the game, put the two pieces side by side on the starting line.

Object: The object of the game is to get to the finish line first.

Game Play: To start the game, decide which player will flip the coin first. Then the first player should flip the coin. If you get heads, then the red car will move one space forward. If you get tails, then the blue car will move one space forward. Now the second person will flip the coin. Again, if you get heads, then the red car will move one space forward. If you get tails, then the blue car will move one space forward. Continue playing the game until you reach the finish line. Once the game is over, record the winner and loser on the tally chart handout. Repeat the game until you have the first three rows of the chart complete.

Directions for using the TI 83 or TI 84 graphing calculator to simulate a coin toss

Tossing a fair coin (equal probability of getting heads or tails)

1. Use the APPS button (it is the blue button)
2. Scroll down to Prob Sim (or use the ALPHA button and 8), enter
3. Enter
4. Choose toss coin
5. Pressing the zoom button will get you into setup to change option, use the arrow buttons to change what you want to change. Press the graph key when you are finished. (the first row of keys are the ones to use for the various options)
6. Press the window key to toss 1 coin
7. Press the +10 or + 50 option to continue tossing the coin

To toss a weighted coin (unequal probability of getting heads or tails)

1. Follow steps 1 to 5 above
2. When you are in the setup option use the window button to go to advanced options.
3. Change the weighting to what you want by using the arrows
4. Press the graph button to exit and save
5. Proceed as above

Results Handout

GROUP RESULTS	Outcomes	WINNER
Game 1		
Game 2		
Game 3		
CLASS RESULTS		
Game 4		
Game 5		
Game 6		
Game 7		
Game 8		
Game 9		
Game 10		
Game 11		
Game 12		
Game 13		
Game 14		
Game 15		
Game 16		
Game 17		
Game 18		
Game 19		
Game 20		

Experimental Probability

Red Car Winning: _____

Blue Car Winning: _____

Tie: _____

Heads on Flip 1: _____

Tails on Flip 1: _____

$$\text{Experimental Probability} = \frac{\text{Number of Times Event Occurs}}{\text{Number of Times the Experiment is Conducted}}$$

Discussion Questions

Use the Tree Diagram Handout to answer the following questions.

1. How many possible outcomes are there for this activity?

2. How many outcomes result in heads winning?

3. How many outcomes result in tails winning?

4. How many outcomes result in a tie?

5. Why is this a fair game? Do you have an equally likely chance of flipping a heads or a tails on each flip?

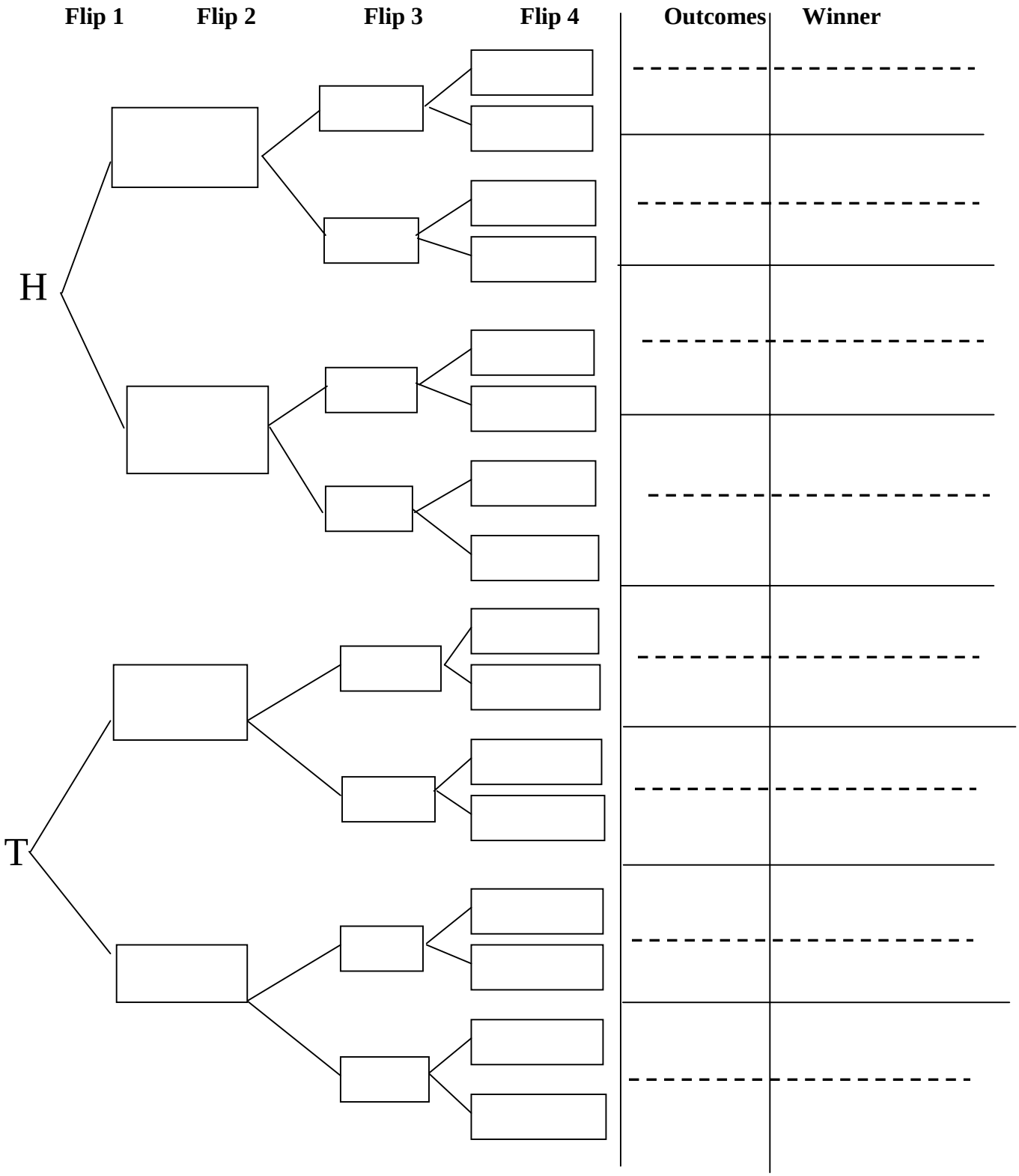
Theoretical Probability = $\frac{\text{number of outcomes that makes the event happen}}{\text{total number of possible outcomes}}$

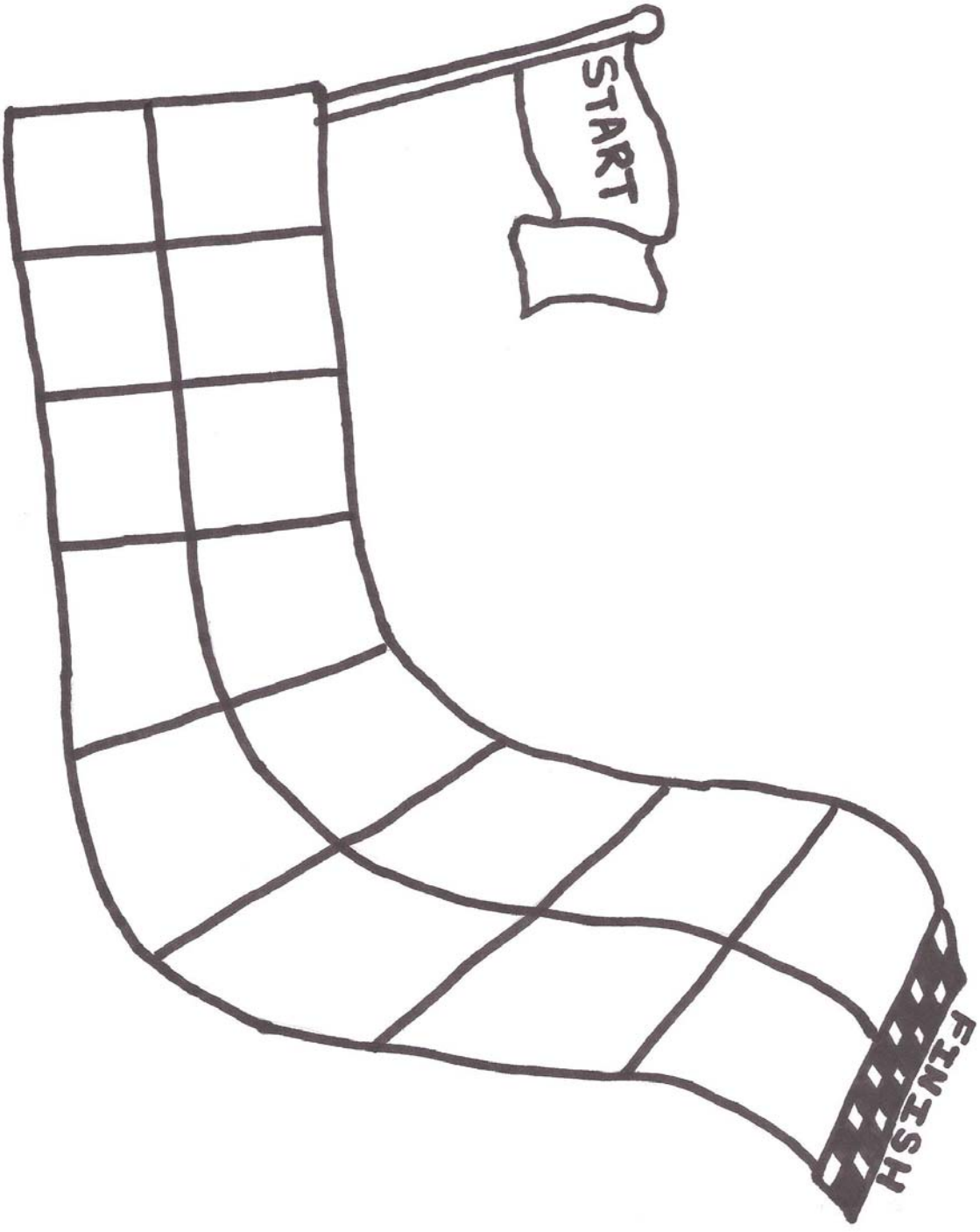
6. What is the theoretical probability that on flip 1 you will flip a heads, a tails?

7. What is the theoretical probability of winning if you choose heads for every flip? What is the theoretical probability of winning if you choose tails for every flip? What is the theoretical probability of having a tie?

8. Describe how your answers in question 6 compare with the class experimental probabilities of flipping a heads on flip 1. (The 1st flip in each game.) Refer to the data you recorded in the Results Handout. Why would the experimental and theoretical probabilities be different?

Tree Diagram Handout for computing theoretical probability

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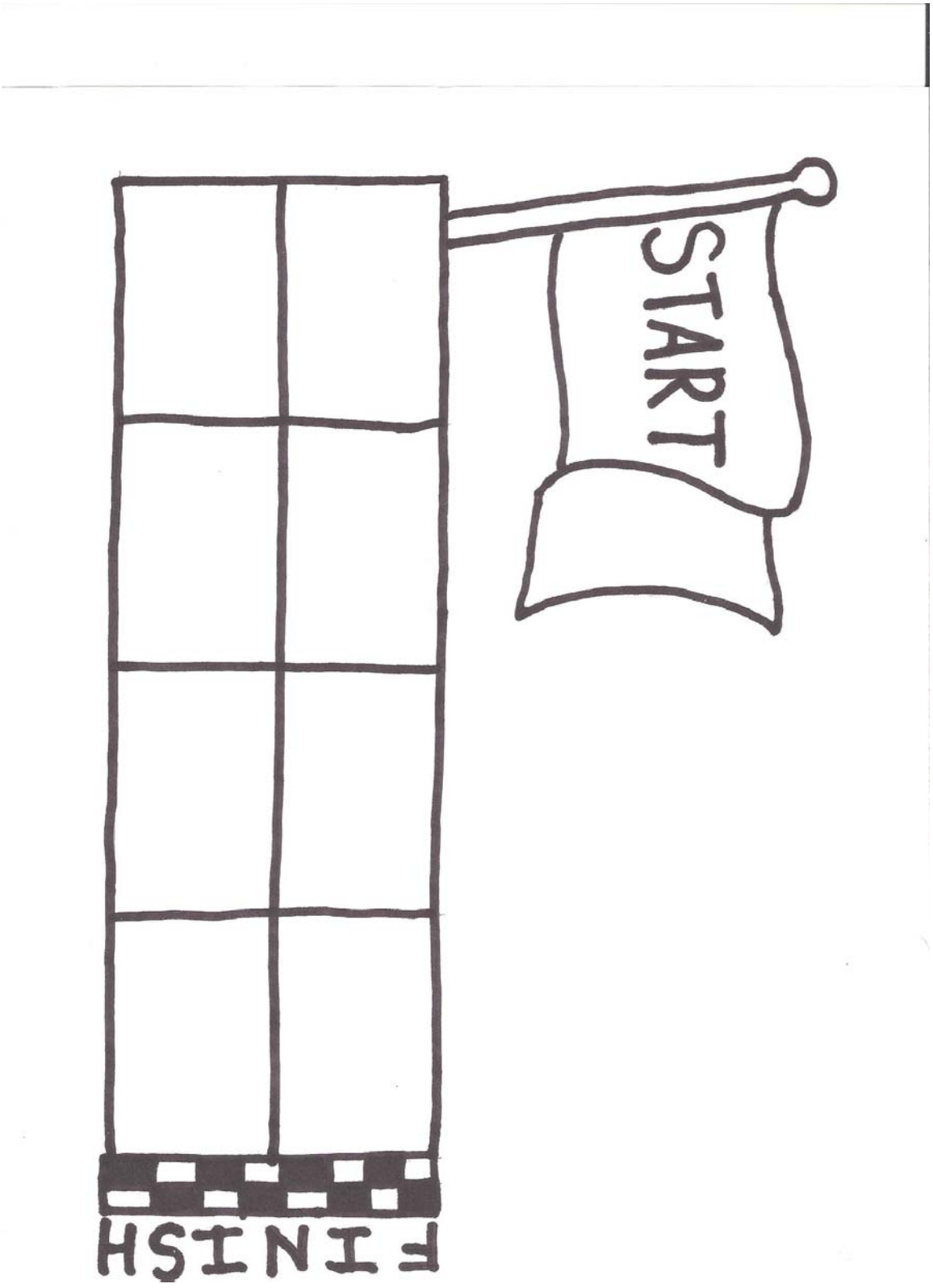
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